Math 516 **Fall 2008** **Radford**

Written Homework # 3

Due at the beginning of class 10/24/08

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1. Let G be a group of order $p^n q$, where p, q are primes and $p^n = q+1$. Show that G is not simple.
2. Let G be a group of order pqr, where p, q and r are distinct primes. Show that G is not simple.
3. Use Sylow's Theorem (§4.5) to prove Cauchy's Theorem (§3.2).
4. Let $G = S_n$, where $n \geq 5$, and $H \leq G$ satisfies $|G : H| < n$. Show that $H = G$ or $H = A_n$.
5. Let G_1, G_2 be groups. A *product of* G_1 *and* G_2 is a triple (P, π_1, π_2), where

(P1) P is a group and $\pi_i : P \longrightarrow G_i$ is a homomorphism for $i = 1, 2$;

(P1) If (P', π'_1, π'_2) satisfies (P1) then there is a homomorphism $f : P' \longrightarrow P$ determined by $\pi_i \circ f = \pi'_i$ for $i = 1, 2$.

Show:

(a) There is a product (P, π_1, π_2) of G_1 and G_2;

(b) If (P, π_1, π_2) and (P', π'_1, π'_2) are products of G_1 and G_2 there is an isomorphism $f : P \longrightarrow P'$.