MATH 431 # Written Homework 11 Solution **Radford**

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1. Page 540, number 2: (**30 points**) 2 (**10**), 3 (**10**), 3 (**10**) respectively.

2. Page 540, number 4: (**30 points**) Write $u = (a_1, \ldots, a_n)$ and $v = (b_1, \ldots, b_n)$. Then $d(u, v) = |\{i \,|\, 1 \leq i \leq n,\, a_i \neq b_i\}|$ and thus

$$d(u, v) = n - |\{i \,|\, 1 \leq i \leq n,\, a_i = b_i\}|. \tag{1}$$

(a) For $1 \leq i \leq n$, $a_i = b_i$ if and only if $b_i = a_i$. Therefore $d(u, v) = d(v, u)$ by (1) (**10**).

(b) By (1) observe that $d(u, v) = 0$ if and only if $|\{i \,|\, 1 \leq i \leq n,\, a_i = b_i\}| = n$ if and only if $a_i = b_i$ for all $1 \leq i \leq n$ if and only if $u = v$ (**10**).

(c) Write $w = (c_1, \ldots, c_n)$. Then $u+w = (a_1+c_1, \ldots, a_n+c_n)$ and $v+w = (b_1+c_1, \ldots, b_n+c_n)$. Let $1 \leq i \leq n$. Since $a_i + c_i = b_i + c_i$ if and only if $a_i = b_i$, $d(u + w, v + w) = d(u, v)$ by (1) again (**10**).

3. Page 541, number 14: (**40 points**) Let $F = \mathbf{Z}_2$ and $V \subseteq F^n$ be the binary code. Fix $1 \leq i \leq n$. Then the map $\pi : V \longrightarrow F$ defined by $\pi(a_1, \ldots, a_n) = a_i$ is a homomorphism of additive groups. Note that

$$\mathrm{Ker}\pi = \{(a_1, \ldots, a_n) \in V \,|\, a_i = 0\}$$

and thus consists of the elements of V whose i^{th} component is 0 (**10**).

Suppose that $\mathrm{Im}\pi = (0)$. Then $\mathrm{Ker}\pi = V$ which means that all code words (elements of V) have 0 in the i^{th} component (**10**).

Suppose that $\mathrm{Im}\pi \neq (0)$. Then $\mathrm{Im}\pi = F$ as the latter has 2 elements. Since $V/\mathrm{Ker}\pi \simeq F$ by the First Isomorphism Theorem for groups (**10**), the calculation $|V| = [V : \mathrm{Ker}\pi]|\mathrm{Ker}\pi| = 2|\mathrm{Ker}\pi|$ shows that half of the element of V have 0 in the i^{th} component (**10**).