MATH 215 **Written Homework 4 Solution** **07/10/2009**

1. (**20 points total**) If $n = 1$ the left hand side of the equation is $A \cap A_1$ which is the right hand side. Therefore the equation is true for $n = 1$. (**3**)

Suppose $n \geq 1$ and the equation holds for all sets $A_1, \dots, A_n$ and let $A_1, \dots, A_{n+1}$ be sets. The equation holds for $n = 2$ (this we are allowed to assume). Thus we may assume $n \geq 2$. (**3**) Using the fact that the equation holds for $n = 2$ and the induction hypothesis we calculate

$$\begin{aligned} A \cap (A_1 \cup \cdots \cup A_{n+1}) &= A \cap ((A_1 \cup \cdots A_n) \cup A_{n+1}) \quad (\mathbf{3}) \\ &= (A \cap (A_1 \cup \cdots \cup A_n)) \cup (A \cap A_{n+1}) \quad (\mathbf{3}) \\ &= ((A \cap A_1) \cup \cdots \cup (A \cap A_n)) \cup (A \cap A_{n+1}) \quad (\mathbf{3}) \\ &= (A \cap A_1) \cup \cdots \cup (A \cap A_{n+1}) \quad (\mathbf{3}) \end{aligned}$$

which means that the equation holds for $A_1, \dots, A_{n+1}$. By induction the equation holds for all sets $A_1, \dots, A_n$, where $n \geq 1$. (**2**)

2. (**20 points total**) a) $P(\emptyset) = \{\emptyset\}$ (**5**) b) $P(\{41\}) = \{\emptyset, \{41\}\}$ (**5**) c) $P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$ (**5**) and d) $P(\{\pi, e\}) = \{\emptyset, \{\pi\}, \{e\}, \{\pi, e\}\}$ (**5**).

3. (**20 points total**) We first complete the truth table

$x \in A$	$x \in B$	$x \in A - B$	$x \in A \cap B$	$x \in (A - B) \cap (A \cap B)$	$x \in A$
T	T	F	T	F	T
T	F	T	F	F	T
F	T	F	F	F	T
F	F	F	F	F	F
		(**2**)	(**2**)	(**2**)	(**2**)

.

From columns 3 and 4 we conclude that $x \in A - B$ or $x \in A \cap B$ if and only if $x \in A$. Hence $(A - B) \cup (A \cap B) = A$. (**6**) Since column 5 consists of all F's we conclude that $(A - B) \cap (A \cap B) = \emptyset$. (**6**)

4. (**20 points total**) The sets $(A \cup B)^c$ and $A^c \cap B^c$ are equal since the 4^{th} and 7^{th} columns of the truth table below are the same. (**5**)

$x \in A$	$x \in B$	$x \in A \cup B$	$x \in (A \cup B)^c$	$x \in A^c$	$x \in B^c$	$x \in A^c \cap B^c$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T
		(**3**)	(**3**)	(**3**)	(**3**)	(**3**)

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5. (**20 points total**) We apply $A^c \cap B^c = (A \cup B)^c$ to A^c and B^c and deduce

$$\begin{array}{ccccccc} (A \cap B)^c & = & (A^{cc} \cap B^{cc})^c & = & (A^c \cup B^c)^{cc} & = & A^c \cup B^c. \\ & (\mathbf{6}) & & (\mathbf{7}) & & (\mathbf{7}) & \end{array}$$