Math 215	**Summer 2009**	**Radford**

Written Homework # 7

Due at the beginning of class 07/31/2009

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1. Committees of 7 are to be formed from a group of 11 individuals. In this problem binomial symbols must be computed.

(a) How many such committees are there?

(b) How many committees of 7 individuals can be formed from the 11 if two particular individuals are to be *included* on the committee?

(c) How many committees of 7 individuals can be formed from the 11 if two particular individuals are to be *excluded* from the committee?

(d) How many committees of 7 individuals can be formed from the 11 if one particular individual is to be *excluded* from the committee?

(e) Use parts (c) and (d) and the inclusion-exclusion principle to find the number of committees of 7 individuals which can be formed from the 11, given at least one of two particular individuals is to be excluded.

2. In the following describe functions by tables $\begin{array}{r|c} x & \cdots \\ \hline f(x) & \cdots \end{array}$.

(a) List the isomorphisms $f : \{a, b, c, d\} \longrightarrow \{a, b, c, d\}$ such that $f(a) = c$ and indicate the inverse of each.

(b) List the surjections $f : \{\pi, e, 19\} \longrightarrow \{c, x\}$.

(c) List the injections $f : \{c, x\} \longrightarrow \{\pi, e, 19\}$.

3. In a small town all residents carry only certain denominations of money with them, \$1, \$5, \$10, \$20, \$50, \$100 or \$500. (Carrying no money is a possibility.)

(a) How large must the population be in order to guarantee that at least two residents are carrying the same number of denominations?

(b) How large must the population be in order to guarantee that at least two residents are carrying the same types of denominations?

4. Let $\mathbf{O}^+$ be the set of positive odd integers. Show that $f : \mathbf{Z} \longrightarrow \mathbf{O}^+$ given by $f(n) = \begin{cases} 4n - 1 & : \ n \geq 1 \\ -4n + 1 & : \ n \leq 0 \end{cases}$ is a bijection.

[You may assume basic facts about the integers, in particular any integer n can be written $n = 2m$ or $n = 2m + 1$ for some integer m, but not both, and in either case the integer m is unique.

The following indicates the bijection in concrete terms.

n	$\cdots$	-5	-4	-3	-2	-1	0	1	2	3	4	5	$\cdots$
$f(n)$	$\cdots$	21	17	13	9	5	1	3	7	11	15	19	$\cdots$

5. This exercise is about the inclusion-exclusion principle.

(a) Suppose that $A, B \subseteq U$, where U is a universal set, $|U| = 23$, $|A| = 9$, $|B| = 5$, and $|A \cap B| = 2$. Find $|A^c \cap B^c|$.

(b) Suppose that each tile in a collection of 22 is a square or a circle and is also red or green. Suppose further that there are 9 square tiles, 14 red ones, and 6 which are both square and green. Use the principle of inclusion-exclusion to determine:

(i) the number of tiles which are square or green;

(ii) the number of tiles which are circles and red;

(iii) the number of tiles which are circles or red.