Stat/Econ 473 Game Theory
Problem Set 11

Due: Thursday April 21

1) With probability 3/4 a trainee (Player 1) is lazy (L) and with probability 1/4 she is industrious (I). The trainee knows if she is lazy or industrious, but a potential employer does not. The trainee chooses to work for 40 hours, 60 hours or 80 hours. The cost to the trainee of working is given in the following table

Type	**40hr**	**60hr**	**80hr**
Lazy	50	75	120
Industrious	30	50	80

At the end of this period an employer (Player 2), who knows how many hours the employee worked, decides whether to hire (H) the employee or fire (F) them. If hired the employee gets a paid of 130 while if fired they get a paid 70 (of course, for their net payoff you have to subtract the costs).

The employer's payoff is 40 from hiring an industrious worker, 0 from hiring no one and -35 from hiring a lazy worker.

a) Draw the game tree, including all nontrivial information sets.

For each of the following strategy profiles decide if there is a perfect Bayesian equilibrium with this strategy profile. If so specify the beliefs that make this a perfect Bayesian equilibrium and in all cases justify your answers. When you find an equilibrium is it pooling or separating?

b) A lazy trainee works 40 hours and industrious trainee works 80 hours. The employer hires a trainee who works 80 hours but does not hire a trainee who works 40 or 60 hours.

c) A lazy trainee works 40 hours and industrious trainee works 60 hours. The employer hires a trainee who works 60 or 80 hours but does not hire a trainee who works 40 hours.

d) Both types of trainee work 60 hours. The employer hires a trainee who works 60 or 80 hours but does not hire a trainee who works 40 hours.

e) Both types of trainee work 40 hours. The employer hires no trainees no matter how many hours they worked.

2) With probability r where $0 < r < \frac{3}{5}$ Sadam Hussein (Player 1) has weapons of mass destruction (W) or not (NW). Saddam Huessein, knowing if he has weapons decides to allow UN inspectors (U) or not (N).

President Bush (Player 2) then must decide to invade (i) or not (n). If Sadam Hussein has allowed inspectors, then Bush knows whether or not there are weapons of mass destruction if there are no inspections all Bush knows is that there have been no inspections.

Assume the payoffs

	Hussein	**Bush**
W,U,i	1	3
W,U,n	3	1
W,N,i	2	3
W,N,n	9	1
U,U,i	1	5
U,U,n	4	9
U,N,i	2	6
U,N,n	8	9

a) Draw the game tree including all information sets

b) Find all pure strategy perfect Bayesian equilibria.

Bonus Find p and q such that there is a perfect Bayesian equilibrium where:

• if there are weapons of mass destruction Hussein does not allow inspections;

• if there are weapons of mass destruction Huessin allows inspectors with probability p;

• if inspections are allowed and there are weapons of mass destruction Bush invades;

• if inspections are allowed and there are no weapons of mass destruction Bush does not invade;

• if inspections are not allowed Bush invades with probability q.

3) A new restaurant is high quality (H) or low quality (L) with probability 1/2. The restaurant knows its quality and decides whether or not to advertise (A) or not (N). If they advertise they will spend a on advertising. A consumer does not know the quality of the restaurant but knows whether or not the restaurant has advertised. The consumer then decides to patronize (p) or avoid (a) the restaurant.

If the consumer goes patronizes the restaurant the payoffs (not counting advertising) are

Quality	**restaurant**	**customer**
Low	15	-20
High	30	70

Of course a must be subtracted from the restaurants payoff if they choose to advertise.

If the customer avoids the restaurant the customer's payoff is 0 and the restaurant's payoff is 0 if they did not advertise and $-a$ if they did.

a) Draw the game tree including all information sets.

b) Find a separating perfect Bayesian equilibrium. For what range of values of a does this work?

c) Find a pooling perfect Bayesian equilibrium. For what range of values of a does this work?