MTHT 530 Analysis for Teachers II
Study Guide on Series

The webpage
    http://www.math.uic.edu/∼marker/mtht530/concepts.html
contains a summary of all key concepts and results we have discussed in the
course.

Sample Questions on Series and Sequences of Functions

Consult Study Guides for Midterms 1 and 2 for sample questions on ear-lier course material

1) Let $f_n : (0, 1) \to \mathbb{R}$ be a sequence of functions.
    a) Define $(f_n)_{n=1}^{\infty}$ converges pointwise to f.
    b) Define $(f_n)_{n=1}^{\infty}$ converges uniformly to f.

2) State the Weierstrass M-test.

3) Sketch the proof that every Cauchy sequence converges.
4) Decide if the following are true or false. If False, give a counterexample.
    a) If $\sum_{n=1}^{\infty} a_n$ converges, then so does $\sum_{n=1}^{\infty} a_n^2$.
    b) If f_n converges to f pointwise and f is bounded, then so is f.

5) Decide if the following series converge or diverge. Justify your answers
    a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n}$
    b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n \ln(n)}$

6) Let $f_n : 0 \to \infty$ be the $f_n(x) = \frac{e^{-x^2}}{n}$
    a) Find $\lim_{n \to \infty} f_n$.
    b) Does the sequence converge uniformly?
7) a) Prove that $f(x) = \sum_{n=1}^{\infty} \frac{\sin(nx)}{n^3}$ converges uniformly on $\mathbb{R}$.
    b) Is f continuous?
    c) Is f differentiable?
Justify your answers.