MTHT 530 Analysis for Teachers II
Problem Set 8

Due: Wednesday March 29

Do Problems 15 and 19 from Chapter 14 of Spivak's *Calculus*

1) Suppose there are $M, m > 0$ such that $m \leq f(x) \leq M$ for all $x \in [a, b]$ and f is integrable. Prove that $\frac{1}{f}$ is integrable.

2) Prove that

$$\frac{1}{3\sqrt{2}} \leq \int_0^1 \frac{x^2}{\sqrt{1+x^2}}\, dx \leq \frac{1}{2}.$$

[Hint: Use Exercise 13b) from Chapter 13 of Spivak–we proved this in class–for carefully chosen functions.]