

MTHT 530 Analysis for Teachers II
Problem Set 5

Due: Wednesday February 22

1) Let $r \in \mathbb{R}$. Prove that the polynomial $x^3 - 3x + r$ does not have two roots in the interval $[0, 1]$.

2) Let $f(x) = \sqrt{1+x}$. Use the Mean Value Theorem to prove that

$$f(x) < 1 + \frac{x}{2}$$

for all $x > 0$.

3) Suppose h is differentiable on $[0, 3]$, $h(0) = 1$ and $h(1) = 2$ and $h(3) = 2$.

a) Argue there is $d \in [0, 3]$ with $h(d) = d$.

b) Argue that at some point c we have $h'(c) = \frac{1}{3}$.

c) Argue that $h'(b) = \frac{1}{4}$ at some $b \in [0, 3]$.

4) Use the tangent approximation to $f(x) = \sqrt{x}$ to estimate $\sqrt{4.1}$ and give a bound on the error of your approximation.