MTHT 530 Analysis for Teachers II
Problem Set 12

Due: Wednesday April 26

1) Let $f_n : (0,2) \to \mathbb{R}$ be $f_n(x) = \dfrac{nx}{1+nx^2}$.

a) Find the pointwise limit of (f_n) for $x \in (0,2)$.

b) Is the convergence uniform?

c) Is the convergence uniform on $(1,2)$? erges uniformly on $(1,+\infty)$.

2) Let $f_n(x) = \dfrac{x}{1+nx^2}$. Find the points where f_n attains its maximal and minimal values. Prove that (f_n) converges uniformly. What is the limit function?

3) For each $n \in \mathbb{N}$ let $f_n : \mathbb{R} \to \mathbb{R}$ by

$$f_n(x) = \begin{cases} 1 & \text{if } |x| \geq 1/n \\ n|x| & \text{if } |x| < \frac{1}{n} \end{cases}.$$

a) Find the pointwise limit of f_n.

b)Construct a sequence of continuous functions on $[-5,5]$ that coverges pointwise to a limit function that is unbounded on the set.

4) Decide if the following are true or false. If true, give a proof, if false give a counterexample.

a) If $f_n \to f$ pointwise on $[0,1]$, then $f_n \to f$ uniformly.

b) If $f_n \to f$ uniformly on A and g is bounded on A, then gf_n converges uniformly to gf.

c) If $f_n \to f$ uniformly and each f_n is bounded on A, then f is bounded on A.

d) If $f_n \to f$ uniformly on an interval and each f_n is strictly increasing, then f is also strictly increasing.

e) If $f_n \to f$ pointwise on an interval and each f_n is nondecreasing, then f is also nondecreasing.

5) Let

$$g_n(x) = \frac{nx + x^2}{2n}$$

and set $g(x) = \lim g_n(x)$. Show that g is differentiable in two ways:
a) Compute $g(x)$ and then find g'.

b) Compute $g_n'(x)$ and show they coverge uniformly on every interval $[-M, M]$.