Math 215: Introduction to Advanced Mathematics
Problem Set 10

Due Wednesday November 22

1) A standard deck of cards has 4 suits and each suit has 13 cards 2, 3, ..., 10, J, Q, K, A. In draw poker your are dealt 5 cards.

a) How many 5 card poker hands are there? [Note: The order you are dealt the cards doesn't matter. If you are dealt the A$\heartsuit$, 5$\diamondsuit$, A$\spadesuit$, K$\spadesuit$, and then the 3$\clubsuit$, you have the same hand as if you were dealt 5$\diamondsuit$, K$\spadesuit$, 3 $\clubsuit$, A$\heartsuit$, and then the A$\spadesuit$.]

b) A *flush* is when all 5 cards are from the same suit. How many ways are there to be dealt a flush.

To calculate the probablity of being dealt a flush, divide your answer to b) by your answer to a).

2) Let X be a finite set with $|X| = n$ and let $0 \le r \le n$.

Let $F : \mathcal{P}_r(X) \to P_{n-r}(X)$ be the function $F(A) = X - A$. Prove that X is a bijection and conclude that

$$\binom{n}{r} = \binom{n}{n-r}$$

3) a) Suppose X and Y are disjoint sets. Let

$$\mathcal{A} = \bigcup_{i=0}^{k} \mathcal{P}_i(x)\times\mathcal{P}_{k-i}(Y) = (\mathcal{P}_0(X)\times\mathcal{P}_k(Y))\cup(\mathcal{P}_1(X)\times\mathcal{P}_{k-1}(Y))\cup\ldots\cup(\mathcal{P}_k(X)\times\mathcal{P}_0(Y)).$$

Let $F : \mathcal{A} \to \mathcal{P}_k(X \cup Y)$ be the function

$$F(A \times B) = A \cup B.$$

Prove that F is a bijection.

b) Use a) to conclude that

$$\binom{m+n}{k} = \sum_{i=0}^{r} \binom{m}{i}\binom{n}{r-i}.$$

4) (5pt Bonus) Let X, Y, Z be nonempty sets. Suppose $f : X \times Y \to Z$ and $y \in Y$, let $f_y : X \to Z$ be the function

$$f_y(x) = f(x, y).$$

Define $\Phi : \mathcal{F}(X \times Y, Z) \to \mathcal{F}(Y, \mathcal{F}(X, Z))$ as follows: For $f : X \times Y \to Z$, let $\Phi(f) : Y \to \mathcal{F}(X, Z)$ be the function $y \mapsto f_y$. Prove that Φ is a bijection.

For finite sets X, Y, Z, this shows

$$|Z|^{|X||Y|} = \left(|Z|^{|X|}\right)^{|Y|}.$$