Abstracts of invited talks in the Special Session on Algebraic Logic and Universal Algebra

► K. V. ADARICHEVA AND J. B. NATION, *Largest extension of a finite convex geometry*.
Institute of Mathematics SB RAS, Acad. Koptyug Prosp., 4, 630090, Novosibirsk, Russia.
E-mail: ki13ra@yahoo.com.
Department of Mathematics, University of Hawaii, Honolulu, HI 96822, USA.
E-mail: jb@math.hawaii.edu.

A convex geometry is a closure system with the anti-exchange property. An extension of a convex geometry (X, φ) is another convex geometry (X, ψ), on the same base set X, with the property that the lattice of closed sets of (X, φ) is a sublattice of the lattice of closed sets of (X, ψ). All extensions of a given *finite* convex geometry can be naturally ordered and they form a lattice with respect to this order. The latter fact was formulated implicitly in [1]: when a convex geometry is finite it always has a largest extension. Still, the exact formula for this largest extension was not presented in [1] leaving the question open.

We provide a new proof of the result in [1] that any finite join-semidistributive lattice can be embedded into an atomistic finite join-semidistributive lattice, with the same number of join-irreducible elements. The new construction of such an embedding turns out to be the largest extension, when applied to a finite convex geometry.

[1] K. V. Adaricheva, V. A. Gorbunov, and V. I. Tumanov, *Join-semidistributive lattices and convex geometries*, ***Advances in Mathematics***, vol. 173 (2003), pp. 1–49.

► CLIFFORD BERGMAN, *Boolean semilattices*.
Iowa State University, Ames Iowa 50011, USA.
E-mail: cbergman@iastate.edu.

Let $\mathbf{S} = \langle S, \cdot \rangle$ be a semilattice. The *complex algebra of* $\mathbf{S}$ is the algebra $\mathbf{S}^{+} = \langle P(S), \cap, \cup, \prime, S, \cdot \rangle$ which is the Boolean algebra of all subsets of S augmented with the complex operation given by $X \cdot Y = \{x \cdot y \colon x \in X, y \in Y\}$. The variety of *Boolean semilattices* is generated by all such complex algebras.

The variety of Boolean semilattices is a very rich one, with an interesting arithmetic, many subvarieties and, of course, many open problems. In particular, it is unknown whether the variety is finitely based. In this talk I will survey what little we know about this variety and discuss some of the open problems.

► NIKOLAOS GALATOS, *Varieties of residuated lattices generated by positive universal classes*.
Department of Mathematics, Vanderbilt University, Nashville, TN 37240, USA.
E-mail: ngalatos@math.vanderbilt.edu.

A *residuated lattice* is an algebra $\boldsymbol{L} = \langle L, \wedge, \vee, \cdot, \backslash, /, e \rangle$, such that $\langle L, \wedge, \vee \rangle$ is a lattice, $\langle L, \cdot, e \rangle$ is a monoid, and for all elements $a, b, c \in L$,

$$ab \leq c \Leftrightarrow a \leq c/b \Leftrightarrow b \leq a\backslash c.$$

It is easy to see that residuated lattices form a finitely based variety.

For every positive universal formula in the language of residuated lattices, we construct a variety of residuated lattices, such that a subdirectly irreducible residuated lattice is in the variety iff it satisfies the positive universal formula.

As an application of this correspondence, we prove that the join of two finitely based commutative varieties of residuated lattices is also finitely based.

► PETER JIPSEN, *An online database of classes of algebraic structures*.
Department of Mathematics, Computer Science, Physics, Chapman University, Orange, CA 92866, USA.
E-mail: jipsen@chapman.edu.

Over the course of the previous century, many different types of algebraic structures have been investigated. While much effort has concentrated on the so-called classical structures (groups, rings, fields, vector spaces, modules, lattices, etc.) there has also been a proliferation of other algebraic structures that have been defined and analyzed. Some of them are generalizations or specializations of classical structures, while others are motivated by algebraic versions of logics, or by topology, combinatorics, and computer science. With so many different algebraic structures, it is not a simple task to determine how they are related, and what is currently known about them. General theories like Universal Algebra and Category Theory have done much to provide a useful framework for these investigations.

This talk presents a preliminary version of a database that aims to collect basic facts about classes of algebraic structures and their relationships. While there are several handbooks of algebra and online encyclopedia of mathematics, the aim of this database is to provide an overview of the structures that have appeared in the literature together with links to computational tools that can be used to investigate the structures further. Each database entry for a class of structures has, at minimum, a name, (several equivalent) definition(s), some standard examples, a list of basic properties that hold for all members, and a list of its "nearest" sub- and superclasses. If there are feasible algorithms for deciding syntactic properties of the structures, implementations of such algorithms may be linked to the database, as well as enumerations of some finite members (if any). Most classes are categories in a natural way, and concepts from category theory are used to express structure preserving relationships between different classes. Currently the database contains over a hundred classes from *abelian groups* to *weakly representable relation algebras*.

Recent advances in electronic publishing (MathML, XML style sheet transformation, scalable vector graphics) provide the tools for this project, and will also be discussed briefly. Much work still remains to be done, and if it is successful, this effort will not conclude at some future date, but rather continue to evolve in a collaborative style to ensure that the information is useful and up-to-date.

▸ ÁGNES SZENDREI, *Clones closed under conjugation*.
Bolyai Institute, University of Szeged, Aradi vertanuk tere 1, H-6720 Szeged, Hungary.
E-mail: a.szendrei@math.u-szeged.hu.

Let G be a permutation group acting on a set A. A clone $\mathcal{C}$ of operations on A is called G*-closed* if for every operation $f(x_1,\dots,x_n)$ in $\mathcal{C}$, all conjugates $f^g(x_1,\dots,x_n) = gf(g^{-1}(x_1),\dots,g^{-1}(x_n))$ of f by permutations $g \in G$ also belong to $\mathcal{C}$. Examples of G-closed clones include the clone of all operations, Słupecki's and Burle's clones as well as all clones of homogeneous operations. In the talk we will discuss the following question: for which groups G on a finite set A is the lattice of G-closed clones finite?

The talk will present results of joint work with Keith A. Kearnes.

▸ CONSTANTINE TSINAKIS, *Semi-direct products of residuated lattices*.
Department of Mathematics, Vanderbilt University, Nashville, TN 37240, USA.
E-mail: constantine.tsinakis@vanderbilt.edu.

A *residuated lattice* is an algebra $\boldsymbol{L} = \langle L, \wedge, \vee, \cdot, \backslash, /, e\rangle$ such that $\langle L, \wedge, \vee\rangle$ is a lattice; $\langle L, \cdot, e\rangle$ is a monoid; and the binary operations $\backslash, /$ are the residuals of the monoid multiplication relative to the lattice structure; that is, the following equivalences hold for all $x, y, z \in L$:

$$x \cdot y \leq z \Leftrightarrow x \leq z/y \Leftrightarrow y \leq x\backslash z.$$

The class $\mathcal{RL}$ of all residuated lattices is easily seen to be a finitely based arithmetical variety. It includes algebras that are term equivalent to many well studied and diverse structures, such as generalized Boolean algebras, Brouwerian algebras, relative Stone algebras and lattice-ordered groups. Thus, residuated lattices allow the study of all these algebras under a common language.

The purpose of this talk is to illustrate the importance of semi-direct products in the study of residuated lattices. En route, we use these products to provide simple equational bases for a number of important subvarieties of $\mathcal{RL}$.

This talk will report on joint research with Bjarni Jónsson.

▸ MATT VALERIOTE, *Definable principal congruences and solvability*.
Department of Mathematics and Statistics, Burke Sciences Building 133, McMaster University, Hamilton, Ontario Canada.
E-mail: `matt@math.mcmaster.ca`.

An equationally defined class of algebraic structures $\mathcal{V}$ is said to have Definable Principal Congruences (DPC) if there is a first order formula defining the principal congruences of the algebras in $\mathcal{V}$. In this talk I will present joint work with Pawel Idziak, Keith Kearnes, and Emil Kiss that considers solvable congruences and algebras in a DPC equational class. We find that DPC imposes stronger centrality conditions like nilpotence, or strong abelianness.

▸ GEORGE VOUTSADAKIS, *A result and some questions in CAAL*.
School of Mathematics and Computer Science, Lake Superior State University, 650 W. Easterday Avenue, Sault Sainte Marie, MI 49783, USA.
E-mail: `gvoutsad@lssu.edu`.

Categorical Abstract Algebraic Logic (CAAL) has been proposed in [8], [9], [10] as a generalization of Abstract Algebraic Logic (AAL) [1], [2], [3], [4] and [5], [6], [7] that is appropriate for handling multisignature deductive systems with quantifiers such as equational and first-order logic. The categorical structure of a π-institution plays, in this more abstract framework, the role played by deductive systems in AAL. Also categorical algebra is used in place of universal algebra to handle the possibility of working in categories other than the category of small sets. A recent result [11] on the characterization of algebraizability of π-institutions is presented. This sharpens a result of [8], [9] and provides a perfect analog of a corresponding characterization for the algebraizability of deductive systems in [2]. A few open questions that may lead future research in this area are also proposed.

[1] W. J. BLOK AND D. PIGOZZI, *Protoalgebraic logics*, ***Studia Logica***, vol. 45 (1986), pp. 337–369.

[2] ———, *Algebraizable logics*, ***Memoirs of the American Mathematical Society***, vol. 77, no. 396 (1989).

[3] J. CZELAKOWSKI, *Equivalential logics I, II*, ***Studia Logica***, vol. 40 (1981), pp. 227–236, 355–372.

[4] J. M. FONT AND R. JANSANA, *A general algebraic semantics for sentential logics*, ***Lecture Notes in Logic***, vol. 7 (1996), Springer-Verlag, Berlin Heidelberg 1996.

[5] B. HERRMANN, *Equivalential logics and definability of truth*, ***Dissertation***, Freie Universitat Berlin, Berlin 1993.

[6] ———, *Equivalential and algebraizable logics*, ***Studia Logica***, vol. 57 (1996), pp. 419–436.

[7] ———, *Characterizing equivalential and algebraizable logics by the Leibniz operator*, ***Studia Logica***, vol. 58 (1997), pp. 305–323.

[8] G. VOUTSADAKIS, *Categorical abstract algebraic logic*, ***Doctoral Dissertation***, Iowa State University, Ames, Iowa, August 1998.

[9] ———, *Categorical abstract algebraic logic: equivalent institutions*, (To appear in ***Studia Logica***)

[10] ———, *Categorical abstract algebraic logic: algebraizable institutions*, ***Applied Categorical Structures***, vol. 10 (2002), pp. 531–568.

[11] ———, *Categorical abstract algebraic logic: the criterion for deductive equivalence*, (To appear in ***Mathematical Logic Quarterly***).