Math 210, **First Hour Exam**, Fall 2005 SOLUTIONS

1. (20 points) Consider the triangle with vertices

$$A = (1, -3, -2),\ B = (2, 0, -4),\ C = (6, -2, -5).$$

(a) Find the area of this triangle.

Compute 2 vectors/sides of the triangle: $\overrightarrow{AB} = \langle 1, 3, -2\rangle,\ \overrightarrow{BC} = 5, 1, -3.$

Compute their cross product: $\overrightarrow{AB} \times \overrightarrow{AC} = \langle -7, -7, -14\rangle = -7 \cdot \langle 1, 1, 2\rangle.$

The area is

$$\frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{7\sqrt{6}}{2}.$$

(b) Determine whether or not it is a right triangle.

Check whether ***any*** *of the sides are orthogonal.*

$\overrightarrow{AB} \cdot \overrightarrow{AC} \neq 0 \Rightarrow$ *these two sides are not orthogonal.*

However, $\overrightarrow{BC} = 4, -2, -1$ *and* $\overrightarrow{AB} \cdot \overrightarrow{BC} = 0 \Rightarrow$ *these two sides are.*

2. (15 points) Find an equation for the plane which contains the point $(2, -1, 5)$ and the line

$$\mathbf{r}(t) = \langle 1 + 4t, 4 + 2t, 1 + t\rangle.$$

Take the line's direction vector $\mathbf{a} = \langle 4, 2, 1\rangle$ *and the vector connecting two points on the plane* $\mathbf{b} = \langle 2, -1, 5\rangle - \mathbf{r}(0) = \langle 1, -5, 4\rangle$.

Their cross product $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 13, -15, -22\rangle$ *is a normal vector to the plane.*

Therefore, the equation

$$\mathbf{n} \cdot \langle x - 2, y + 1, z - 5\rangle = 13x - 15y - 22z + 69 = 0$$

is one possible equation of the plane.

3. (15 points) For the position function $\mathbf{r}(t) = \langle t, t^2, \sin(t)\rangle$, find the velocity $\mathbf{v}(t)$, the speed $v(t)$, and the acceleration $\mathbf{a}(t)$.

Velocity: $\mathbf{r}'(t) = \langle 1, 2t, \cos(t)\rangle$.

Speed: $|\mathbf{r}'(t)| = \sqrt{1 + 4t + \cos^2(t)}$.

Acceleration: $\mathbf{r}''(t) = \langle 0, 2, -\sin(t)\rangle$.

4. (15 points) Sketch the level sets for the function $f(x, y) = 4x^2 + 4y^2 + 2$ which correspond to the function values 2, 6, and 18.

The first level curve is $f(x, y) = 4x^2 + 4y^2 + 2 = 2$ *or* $x^2 + y^2 = 0$ *is the origin.*

The second and the third equations, $f(x, y) = 6$ *and* $f(x, y) = 18$ *after simplification become the equations of the circles of radii* 1 *and* 2.

5. (10 points) Evaluate the following limit, or show it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{2x^2 + y^2}$$

The limit along the y*-axis* $x = 0$ *is 0, but the limit along the line* $y = x$*, for example, is* $2/3$. *Therefore, the limit in the question does not exist.*

6. (15 points) For the function $f(x, y) = e^{5x} \cos y$, find the partial derivatives f_y, f_{xy}, and f_{yy}.

$f_y = -e^{5x} \sin y$

$f_{xy} = fyx = \frac{\partial f_y}{\partial x} = -5e^{5x} \sin y$

$f_{yy} = \frac{\partial f_y}{\partial y} = -e^{5x} \cos y$

7. (10 points) Solution discussed in class.