# mpde57: Abstracts

## 57th Midwest Partial Differential Equations Seminar April 21-23, 2006, at UIC

* **Antonio Bove (Bologna)**

TITLE: Analytic Hypoellipticity in the Presence of Lower Order Terms

ABSTRACT: We consider a second order operator with analytic coefficients whose principal symbol vanishes exactly to order two on a symplectic real analytic manifold. We assume that the first (non degenerate) eigenvalue vanishes on a symplectic submanifold of the characteristic manifold. In the $C^\infty$ framework this situation would mean a loss of 3/2 derivatives (as Helffer has shown). We prove that this operator is analytic hypoelliptic.

* **Jonathan Cohen (UIC and DePaul)**

TITLE: Global existence for a coupled system of KdV-like equations with rough initial data

ABSTRACT: This talk concerns coupled systems of Korteweg-deVries type equations. Such systems arise in various contexts including internal wave propagation. We outline a theory of global well-posedness for such systems. This is done in two natural steps- a local well posedness theory set in $H^s(R)$ where $s > -(3/4)$ and a corresponding global extension. The extension relies on approximation of energy inequalities.

* **Luca Capogna (Arkansas)**

TITLE: Generalized mean curvature flow in Carnot group

ABSTRACT: The sub-Riemannian mean curvature flow consists in flowing hypersurfaces in the direction of the "horizontal normal" with velocity depending on the horizontal mean curvature. The presence of so-called characteristic points makes this study much harder than its Riemannian counterpart. The level set formulation of the flow, coupled with the notion of viscosity (or generalized) solution extends well to the sub-Riemannian setting and allows to prove analogues of the results by Evans-Spruck, Chen-Giga-Goto. I will describe ongoing joint work with Giovanna Citti (U. Bologna) where we study viscosity solutions of the mean curvature flow of level sets in Carnot groups.

* **Loredana Lanzani (Arkansas)**

TITLE: Hodge systems and the d-bar problem

ABSTRACT: In the first part of this talk I will give an overview of joint work with E. M. Stein concerning $L^r$–estimates of the Hodge system for forms in $\mathbf{R^N}$. In the second part I will discuss the following question: can these results be used to obtain new estimates of the d-bar problem for $(p, q)$–forms in $\mathbf{C^n}$?

* **Andreea Nicoara (Harvard)**

TITLE: The Kohn Algorithm in More General Classes of Functions

ABSTRACT: In 1979 Joseph J. Kohn introduced an algorithm that yields ideals of subelliptic multipliers, which measure whether there is gain in regularity of the solution of the $\bar{\partial}$–Neumann problem on a pseudoconvex domain in $\mathbf{C^n}$. If such a domain is defined by a real analytic function, subellipticity of the $\bar{\partial}$–Neumann problem is equivalent to the Kohn algorithm generating the entire ring of real analytic functions. I will discuss what happens to this equivalence for local rings of functions that are strictly larger than the local ring of real analytic functions, in particular those of the Denjoy–Carleman classes of functions.

* **Wolfgang Staubach (Chicago)**

TITLE: $\Psi$-pseudodifferential operators and estimates for maximal oscillatory integrals

ABSTRACT: We define a class of pseudodifferential operators with symbols $a(x, \xi)$ without any regularity assumptions in the $x$ variable and explore their $L^p$ and $H^1$ boundedness properties. The results are used to obtain estimates for certain maximal operators associated with oscillatory singular integrals. (Joint work with Carlos Kenig)

* **Jie Shen (Purdue)**

TITLE: Fast Spectral–Galerkin Method: Algorithms, Analysis and Applications

ABSTRACT: In recent years, spectral methods have become increasingly popular among computational scientists and engineers because of their superior accuracy and efficiency when properly implemented. In this talk, I shall present fast spectral–Galerkin algorithms for some prototypical partial differential equations. These spectral-Galerkin algorithms have computational complexities which are comparable to those of finite difference and finite element algorithms, yet they are capable of providing much more accurate results with a significantly smaller number of unknowns. A key ingredient for the efficiency and stability of the spectral–Galerkin algorithms is to use (properly defined) generalized Jacobi polynomials as basis functions.

I shall present applications of these fast spectral—Galerkin algorithms to a number of scientific and engineering problems, including KdV type equations, Navier–Stokes equations, multiphase incompressible flows and bounded obstacle scattering.

* **Catherine Sulem (Toronto)**

TITLE: Water waves over a varying bottom

ABSTRACT: We will discuss the problem of nonlinear wave motion of the free surface of a body of fluid with a varying bottom. The object is to describe the character of wave propagation in a long-wave asymptotic regime for two and three dimensional flows. We consider bottom topography which is periodic in horizontal directions on a short length-scale and may in addition exhibit slow variations. For two dimensional flows, we will also consider the case of a random rough bottom. The purpose is to understand how the varying bottom affects the effective Boussinesq equations and in the appropriate unidirectional limit, KdV or KP type equations.

* **Stephen Yau (UIC and Harvard)**

TITLE: Real time solution of Duncan–Mortensen–Zakai equation in nonlinear filtering

ABSTRACT: It is well known that filtering theory plays an important role both in commercial and military industries. In this talk we shall give a brief introduction of nonlinear filtering. The central problem is to solve Duncan–Mortensen–Zakai (DMZ) equation (which is a time dependent parabolic equation) in real time. We shall describe some previous rigorous methods in solving special DMZ equation. We shall also describe our new general method to solve DMZ equation in real time which is a joint work with S. T. Yau.

* **Huiqiang Jiang and Wei-Ming Ni (Minnesota)**

TITLE: On Steady States of Van Der Waals Force Driven Thin Film Equations

ABSTRACT: Let $\Omega \subset \mathbf{R}^{\mathbf{N}}$, $N \geq 2$ be a bounded smooth domain and $\alpha > 1$. We are interested in the singular elliptic equation

$$\Delta h = \frac{1}{\alpha} h^{-\alpha} - p \text{ in } \Omega$$

with Neumann boundary conditions. We gave a complete description of all continuous radially symmetric solutions. In particular, we constructed nontrivial smooth solutions as well as rupture solutions. Here a continuous solution is said to be a rupture solution if its zero set is nonempty. When $N = 2$ and $\alpha = 3$, the equation has been used to model steady states of van der Waals force driven thin films of viscous fluids. We also considered the physical problem when total volume of the fluid is prescribed.

* **Ivana Alexandrova (Toronto)**

TITLE: The Scattering Amplitude at a Maximum of the Potential

ABSTRACT: We consider the semi-classical scattering amplitude for a short range perturbation of the Laplacian at an energy which is a maximum of the Laplacian. We prove that the scattering amplitude is a semi-classical Fourier integral operator associated to the appropriately defined scattering amplitude.

* **Maxim Zyskin (Bristol)**

TITLE: Liquid Crystals on Polyhedral Domains.

ABSTRACT: We consider director fields (ie maps to $RP^2$) on a contractible polyhedra with tangent boundary conditions on faces. Tangent boundary conditions prevent continuity at vertices; assuming continuity elsewhere, there are many topologically inequivalent configurations, classified by certain homotopy invariants. We establish a lower bound for the Dirichlet energy of such maps as a function of the homotopy invariants. For maps on a rectangular prism, we establish an upper bound which differs from the lower bound by a factor which depends only on the aspect ratios of the prism but not on the invariants. We discuss a smooth-to singular-on edge transition for a certain topologically nontrivial solution on a prism, as a function of prism aspect ratios. This work has applications to new multi-stable nematic liquid crystal devices.

* **Siddhartha P Chakrabarty (UIC)**

TITLE: Cancer drug delivery in three dimensions for a PDE driven model using finite elements

ABSTRACT: The Galerkin finite element method for a three dimensional case in spherical so-ordinates is used to develop procedures for the optimal drug delivery to brain tumors. The mathematical model comprises of a system of three coupled reaction diffusion models, involving the density of tumor cells and normal tissue as also the drug concentration. An optimal control problem is formulated with the goal of minimizing the tumor cell density and reducing the side effects of the drug. The classical method of calculus of variations is used to obtain a coupled system of forward and backward PDE's. Galerkin finite element method is then used to realistically represent the brain structure. The Galerkin ODEs are solved by a combination of Crank–Nicolson and predictor–corrector methods.

* **Alireza Yazdani (Loughborough)**

TITLE: Derivation of the residual free bubbles using the Method of Least Squares

ABSTRACT: The Galerkins method of weighted residuals is based on local approximation of the solution of a given differential equation, where this approximation is obtained from substitution of a piecewise linear interpolation into equation. Nodal values of the piecewise solution are obtained throughout a reduction of the problem to a linear system of equations. The locality is then resolved via an assembly process, which, along with other steps, form up the very concept of Finite Element Method. The so called bubble function method is developed from the idea of enrichment of the interpolation base functions by addition a bubble function, normally coming from an infinite dimensional augmented linear space, so that this bubble function takes zero at element boundaries which in turn, confines the generated approximation error inside the element as well as relaxing the non-homogeneity of the original boundary conditions [1- 2]. Ideally, the bubble function is the analytical solution of the residual differential equation, subject to homogeneous boundary conditions.

However, the analytical solution is hardly obtainable in general, and as is the case in many practical situations, a simple polynomial approximate form is needed for computational purposes. Many people adopt the aforementioned approach to treat differential equations numerically within the context of finite element modelling of physical phenomena. In this work we assume a polynomial form of the bubble function and derive the approximate polynomial form of the practical bubble, using the method of least squares. This method turns out to yield high degree of accuracy, capability of generalization to higher dimensions and non-linear differential equations as well as being computerizable, where suggest a benchmark for dealing with different classes of differential equations.