Notes on Problems and Projects

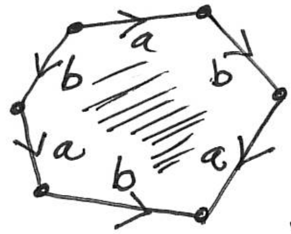
LK

1° Given a finitely presented group G, you can make a 2-diml CW complex $|G|$ with $\pi_1(|G|) \cong G$.

Start with a 1-point join of circles, one for each generator. Add 2-cells, one for each relator.

e.g. $G = (a, b \mid aba = bab)$

$r = abab^{-1}a^{-1}b^{-1}$



add this 2-cell with the indicated boundary identifications.

Since $\pi_1(|G|) \cong G \cong (a, b \mid aba = bab)$, we know that this indentification space is not a 2-manifold (why?).

- Show this directly.
- Examine other examples.
- e.g.

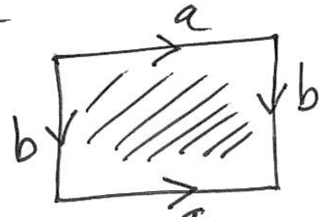
$$\left.\begin{array}{l} b = aba^{-1} \\ a = bab^{-1} \end{array}\right\} \; ab = ba$$

H

$G = \pi_1(S^3 - H) = (a, b \mid ab = ba) \cong \mathbb{Z} \oplus \mathbb{Z}$

$r = aba^{-1}b^{-1}$

Note that

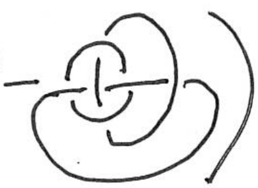


the identification space is a torus. In fact, you can find a torus $\subset S^3 - H$ as a deformation retract.

2.° All finitely presented groups Γ are subject to analysis via the path-lifting Fox calculus + the map

$$\psi : \Gamma \longrightarrow ab(\Gamma).$$

e.g. workout the Alexander module for

a) $\pi_1(S^3 -$  $)$

b) $\Gamma = (a, b \mid ababa = babab)$

c) 

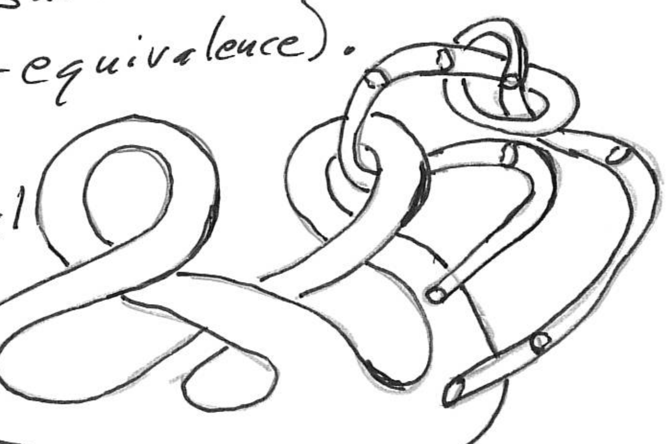
3.° Show that if Θ_F is a Seifert matrix for a surface F with $\partial F = K$ (a given knot) and we define

$$\Delta_K(t) \doteq \mathrm{Det}(\Theta_F - t\,\Theta_F^T)$$

then $\Delta_K(t)$ is invariant up to $\doteq$ if we make a new surface F' by adding a tube (S-equivalence).

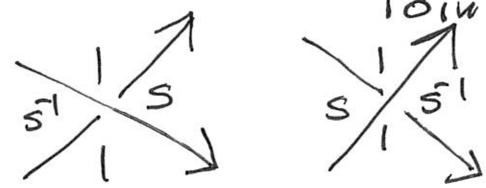
F

F'



4.° Show that the FKT model

$$\nabla_K(z) = \sum_{S \in \text{Pointer (Clock) States}} \langle K|S \rangle (-1)^{b(S)}$$



is invariant under the Reidemeister moves. (Use long knots with standard * placement as in

5.° Investigate sample calculations of $\langle K \rangle$: $\langle \times \rangle = A \langle \asymp \rangle + A^{-1} \langle)(\rangle$

$$\langle O \sqcup K \rangle = \delta \langle K \rangle, \; \delta = -A^{3} - A^{-3}$$

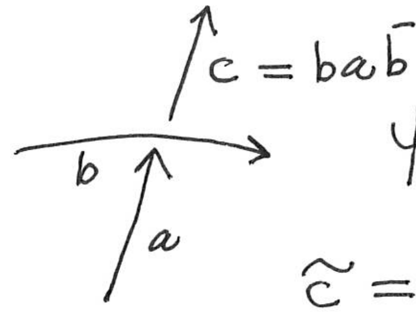
$$\langle O \rangle = 1$$

6.° Investigate sample calculations of Homflypt polynomial.

7. Ditto for Kauffman polynomial.

8. Prove the fraction identities for $F(T) = i \dfrac{\langle N(T) \rangle}{\langle D(T) \rangle}$.

9.°



$$\psi : a \longmapsto t$$
$$b \longmapsto t$$

$$\begin{aligned}
\tilde{c} &= \widetilde{bab^{-1}} \\
&= \tilde{b} + b(\tilde{a} + a\widetilde{b^{-1}}) \\
&= \tilde{b} + b(\tilde{a} + a(-b^{-1})\tilde{b}) \\
&= \tilde{b} + b\tilde{a} + (-bab^{-1})\tilde{b} \\
\tilde{c} &= b\tilde{a} + (1 - bab^{-1})\tilde{b} \\
\psi(\tilde{c}) &= t\tilde{a} + (1-t)\tilde{b}.
\end{aligned}$$

This shows how the quandle structure comes from Fox calculus.

a) Investigate the analogous formalism for the Dehn presentation (one generator per region & one relation per crossing: $\dfrac{A \uparrow D}{B \uparrow C}\rightarrow$: $AB^{-1}CD^{-1} = 1$.)

Here $\psi(\text{region}) = t^{[\text{degree of the region}]}$ where $\psi(\text{outer region}) = t^0$ &

b) Work thru the use of Fox calculus in the paper on our website about Knot Floer Homology. (Grid diagram, minesweeper matrix:

etc.

10° Read the Ken Perko material and write your own description of his calculation of linking numbers in branched covers.

11° Read the material about hyperbolic structures on knot complements. Write your own explanation for the figure eight knot.

12° Read "Quick Trip" by Fox — section on 2-spheres in 4 space. Calculate some Alexander modules for fund grps of $S^2 \hookrightarrow S^4$.