

**Notes and Problems - PMath 766 -Introduction to Knot Theory -
Fall Semester 2003**

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Teaching PMath 766 - Introduction to Knot Theory
Tues and Thursay 1:00PM - 2:30PM
Room: RCH 212

Problems and Notes

These are not systematic notes, just remarks on what we are doing, records
of some problems etc.

1. Prove that in any connected (as a plane graph) shadow diagram D for a knot or link, $V + 2 = R$ where V denotes the number of crossings of the diagram and R denotes the number of regions in the plane delineated by the diagram.
2. Show that a connected 4-regular plane graph D (i.e. a shadow diagram) without multiple edges or loops must have a three sided region.
3. Compute the 3-variable bracket polynomial for the trefoil diagram and for the figure eight diagram. Discuss reconstruction for the polynomial of the figure eight diagram.
4. Write up a discussion in your own words of the analysis of the bracket polynomial behaviour and the invariance under the Reidemeister moves. Give examples and see if you can raise questions that are new for you.
5. Let K_n be the link obtained by taking n half-twists between two parallel strands, and then attaching the initial ends to the final ends. Find recursively a formula for $\langle K_n \rangle$.
6. Prove that $\langle K + K' \rangle = \langle K \rangle \langle K' \rangle$ for any links K and K' , where $K + K'$ denotes the connected sum of K and K' .
7. Do your own classification of all the knots with less than or equal to five crossings and compare your results with those in the knot table from Rolfsen's book.

8. Make a list (using the knot table) of all the knots that are not rational knots with less than or equal to nine crossings.
9. Prove that if K is an alternating diagram and $G = G(K)$ is its checkerboard graph, then if G is equivalent in the two-sphere to its dual graph G^* , then K is ambient isotopic to its mirror image \bar{K} .