Course Outline - PMath 766 -Introduction to Knot Theory - Fall Semester 2003

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Teaching PMath 766 - Introduction to Knot Theory
Tues and Thursay 1:00PM - 2:30PM
Room: RCH 212

Description: This course will introduce the basics of knot theory from a combinatorial point of view. Applications and relationships with statistical mechanics, quantum theory and quantum computing will be included. Along with these applications, an unusual feature of the course will be the inclusion of the theory of virtual knots. Virtual knot theory is a generalization of knot theory based in the structure of the Gauss codes that underly the knot diagrams. The prerequisites for the course are basic abstract algebra and a familiarity with the rudiments of point set topology.

Here is a projected list of topics:

A. Basic Knot Theory

1. Reidemeister moves and the combinatorial (graph theoretic) description of knots and links. Combinatorial description of virtual knot theory.

2. Linking numbers.

3. Spanning surfaces for knots and links, the Seifert pairing and the Alexander polynomial.

4. Invariants of knots derived from coloring, fundamental group and generalizations such as the quandle and the biquandle.

5. The Jones polynomial via the bracket polynomial state model.

6. Applications of the bracket polynomial to alternating and adequate knots and links.

7. Spanning tree expansion of the bracket polynomial and its relationship with the Tutte polynomial.

8. Relationship of the bracket polynomial to the Temperley Lieb algebra and the Potts model in statistical mechanics.

9. Quantum invariants of knots and links. Matrix models for the bracket polynomial and models via solutions to the Yang-Baxter equation. Generalizations of these models to the other invariants of knots, including the Homflypt polynomial, the Kauffman polynomial and the Alexander polynomial.

10. Introduction to Vassiliev invariants (the second term of the course will include more detailed work with Vassiliev invariants).

11. The Khovanov Categorification of the Jones polynomial.

B. Applications

1. Quantum entanglement and topological entanglement: Here we will introduce basic notions of quantum physics and quantum computation, and study the entangling properties of braiding operators from the point of view of topology and from the point of view of quantum physics.

2. Topology of DNA. We will use our work on invariants to classify rational tangles and rational knots and to explain how these topological results are used in the study of DNA recombination. If there is time we will also discuss DNA computing from a knot theoretic point of view.

3. Knots and graphs. (a) Coloring problems for graphs such as the four color problem can be reformulated using language of knot theory. We discuss this using the Temperley Lieb algbra and viaabstract tensors and the Penrose state summation. Other aspects of graph coloring are very close in spirit to knot theoretic ideas and we will explore these as well. (b) Conway and Gordon proved that the complete graph on seven nodes is intrinsically knotted (in the sense that any embedding of this graph in 3-space must contain a knot). We will prove the Conway-Gordon theorem and discuss problems in this area. (c) The Reidemeister moves translate to a set of moves on signed plane graphs that preserve electrical conductivity of the corresponding graph. We will discuss aspects of this relationship.

4. There are other important applications using the interlink between quantum field theory, general relativity, quantum gravity and knot theory that will be discussed in the second semester of this course.

C. Suggested Texts

1. Louis H. Kauffman, "Knots and Physics", World Scientific, Third edition (2002).

2. Prasolov and Sossinsky, "Knots, Links, Braids and 3-Manifolds" AMS Translations of Mathematical Monographs, Volume 154, American Mathematical Society (1997).