Topological Quantum Information Theory

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See

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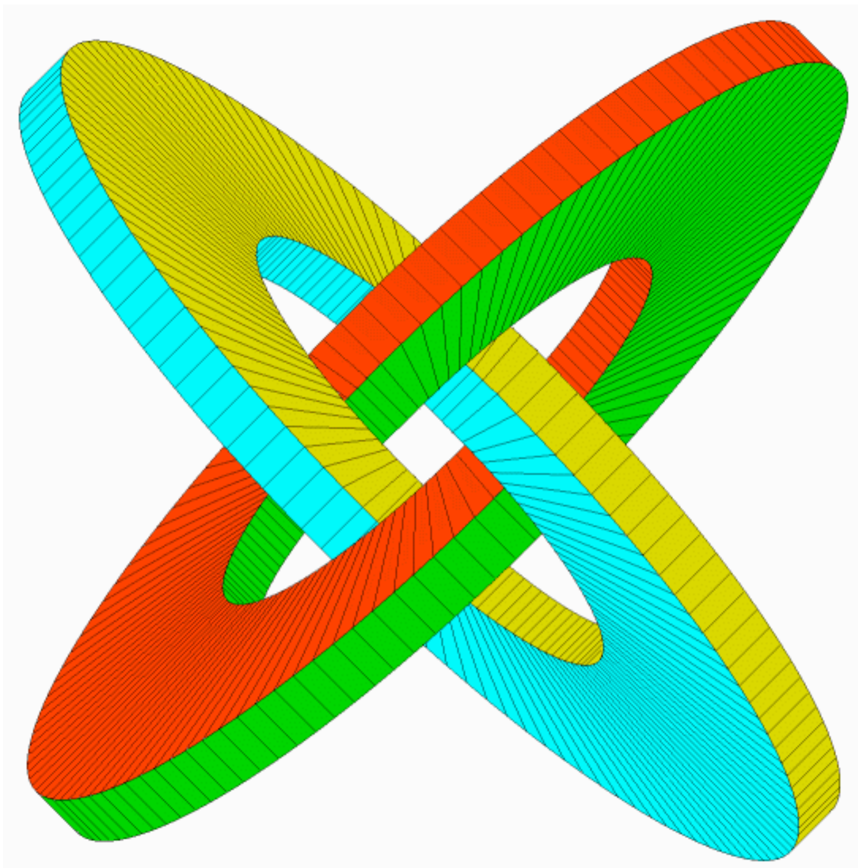
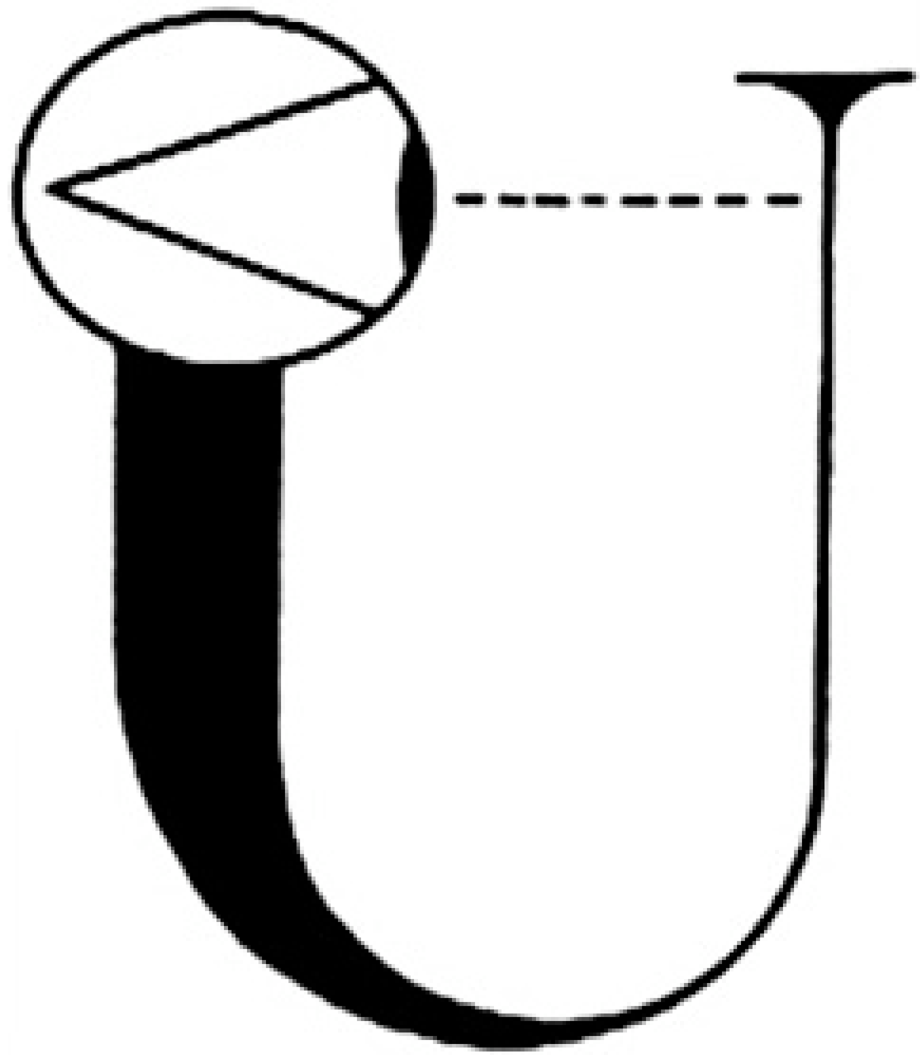
and links therein.

Much of this work is joint work with Samuel J. Lomonaco Jr.

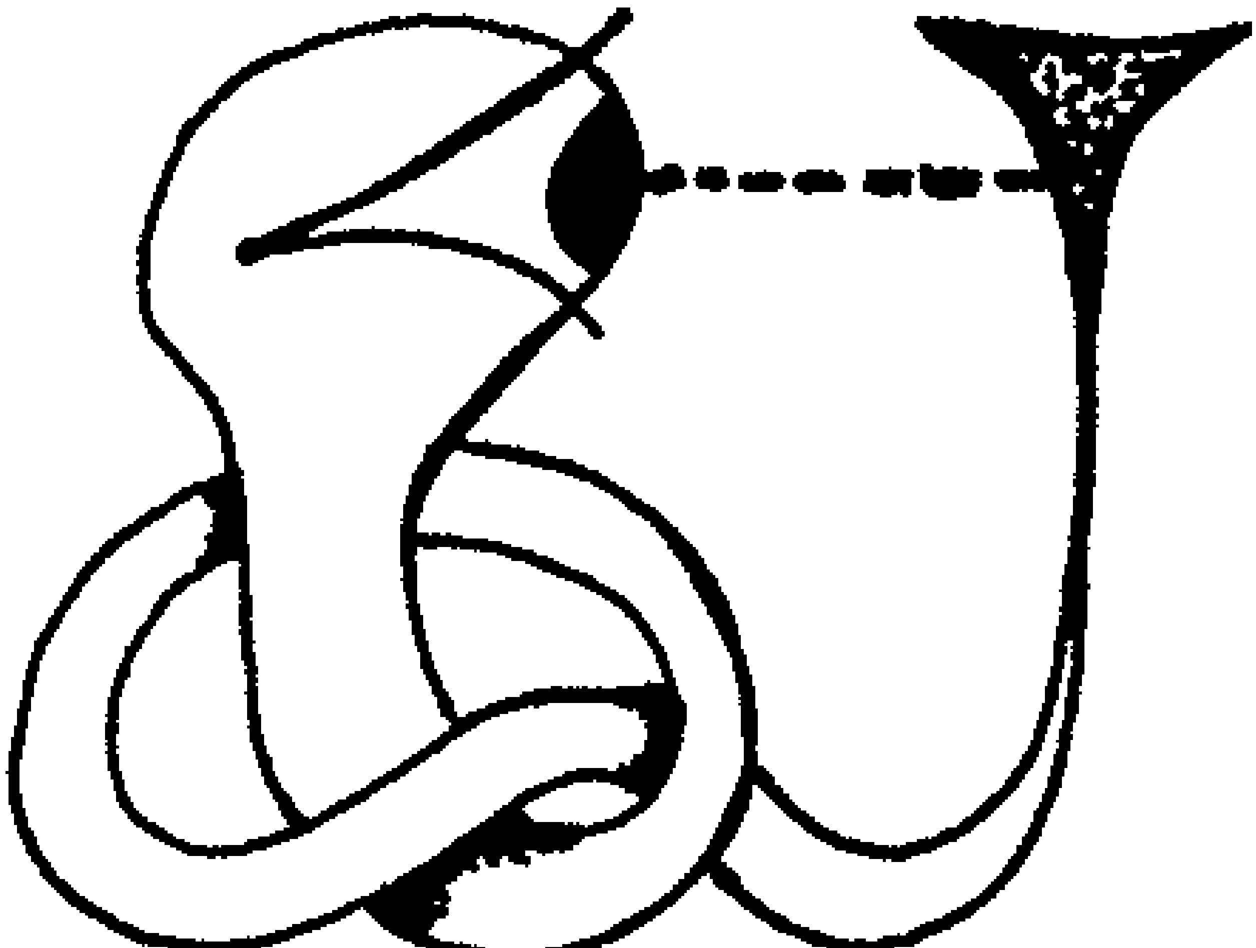


Impossibly linked ambiguous rings. © 2004 by Donald Simanek.

John Wheeler's Universe as Quantum Self-Excited Circuit



Our Universe as a Quantum Knot
Self-Excited Circuit



In the hefty book "Gravitation" by
Misner, Thorne and Wheeler
it is suggested that

Physics should be a manifestation of logic:
Pregeometry as a form of the calculus of
propositions.

This is proposed as an idea for an idea.
We read:
Logic
Icon
Diagrammatic Categories
Knots and Topology.

Theme of Spin Networks

Roger Penrose originally defined SU(2) Spin Networks in a search for a process background for spacetime.

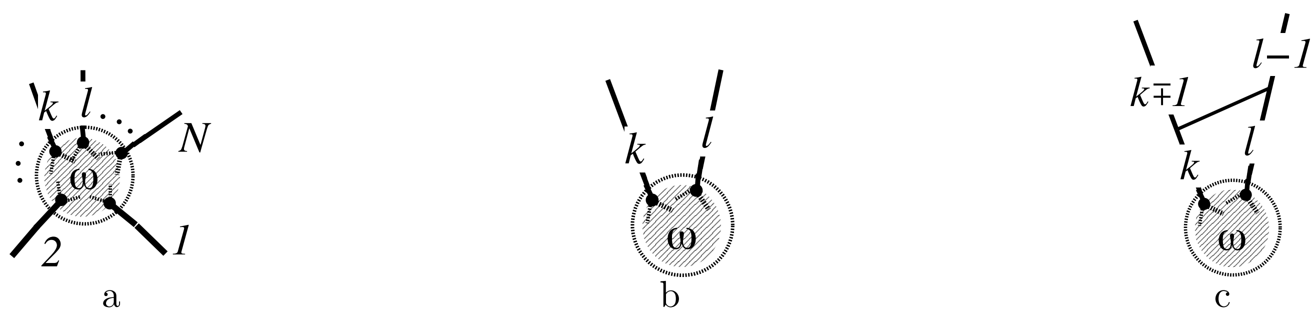
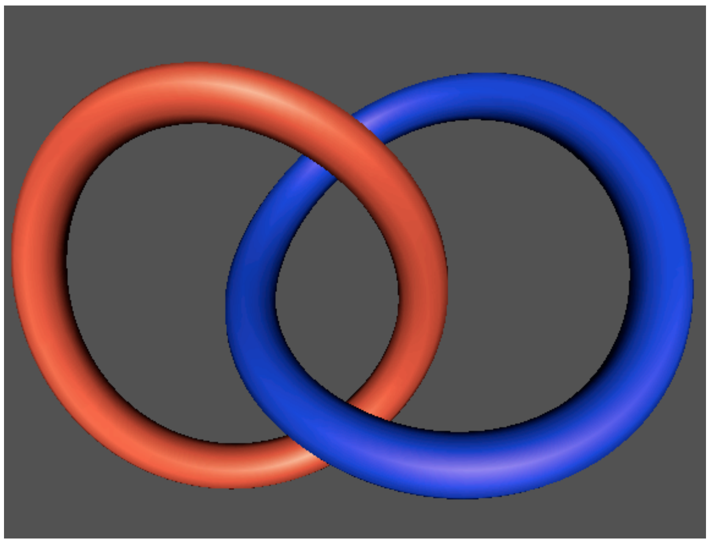
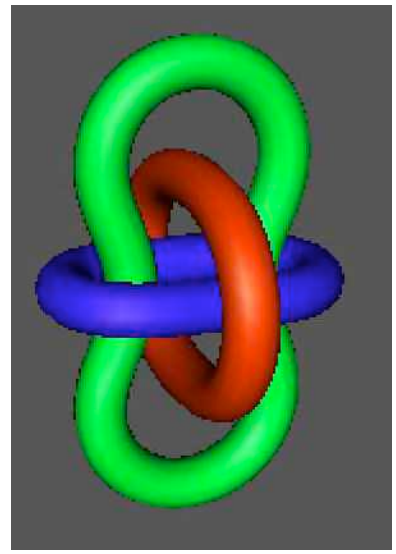


FIGURE 1. (a.) A spin network state with N external lines based on the invariant ω; a spin network with only these N open lines. The lines are labeled $1, 2, \ldots, N$. Two of the spins k and l are identified. (b.) A particular example with two lines of k and l spin. (c.) The exchange of a spin-1/2 "particle." This "experiment" helps determine the angle between the two lines.

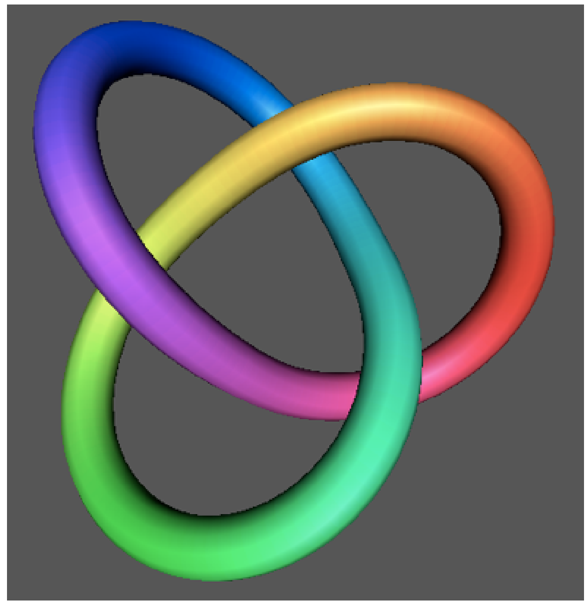
Knot Logic





Linking As Mutuality

Self-Mutuality and Fundamental Triplicity

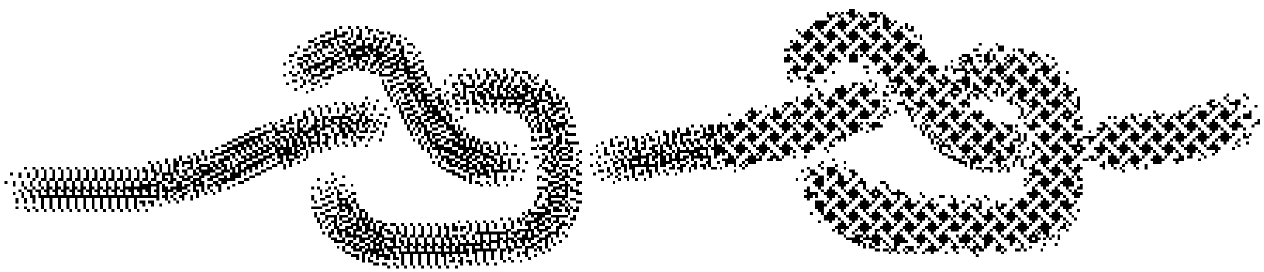


Trefoil as stable self-mutuality
in three loops about itself.

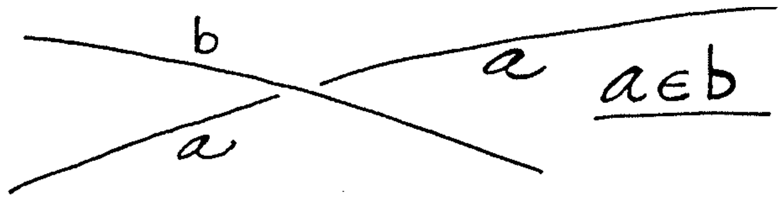
Patterned Integrity

The knot is structurally independent
of the substrate that carries it.

All information in the knot
occurs in its relationship with the ambient space.



Knot Sets



$a \in b$

Crossing as Relationship

$a = \{a\}$

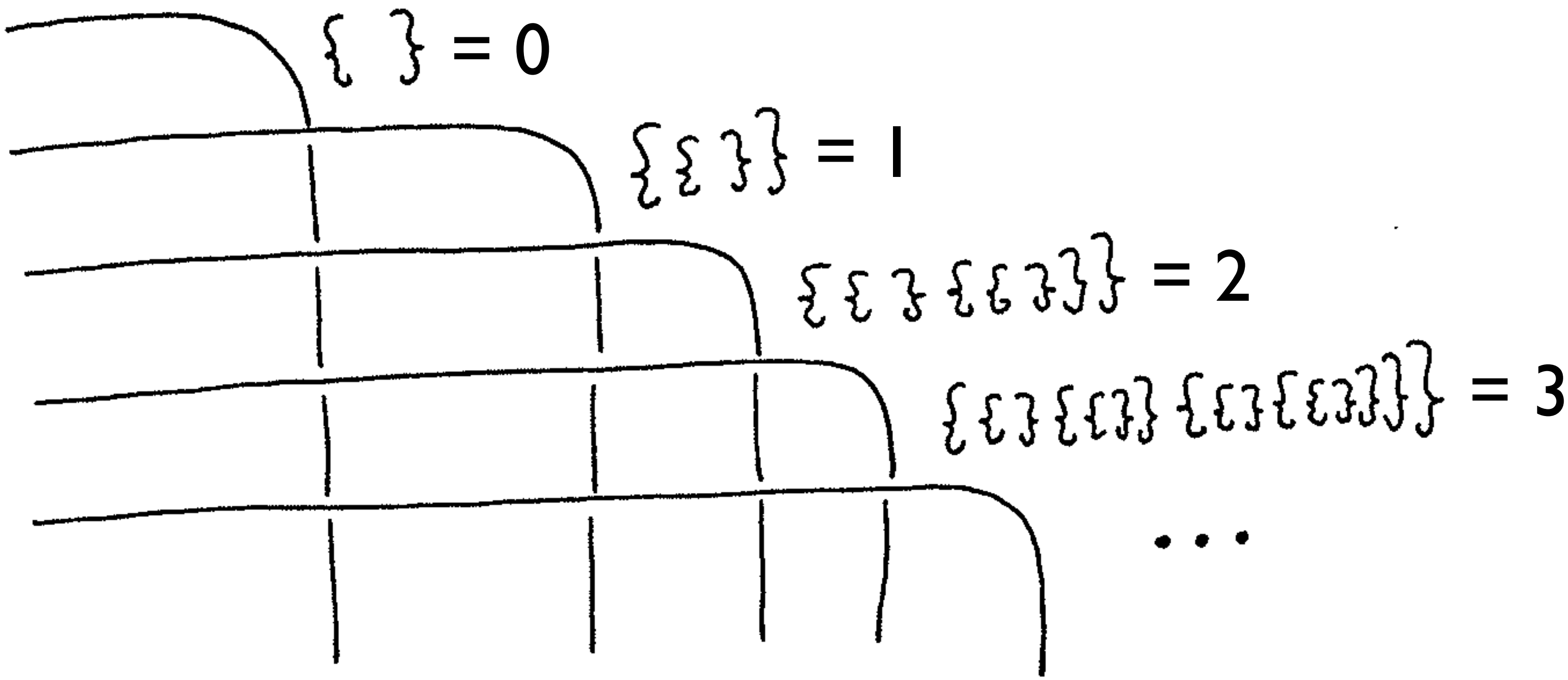
Self-Membership

$a = \{b\}$

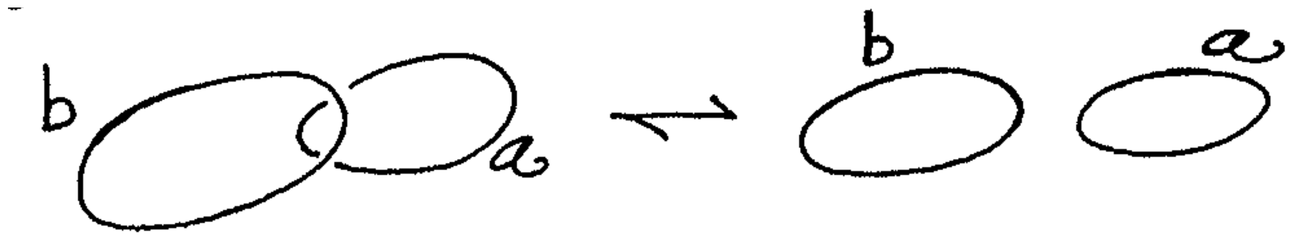
$b = \{a\}$

Mutuality

Architecture of Counting

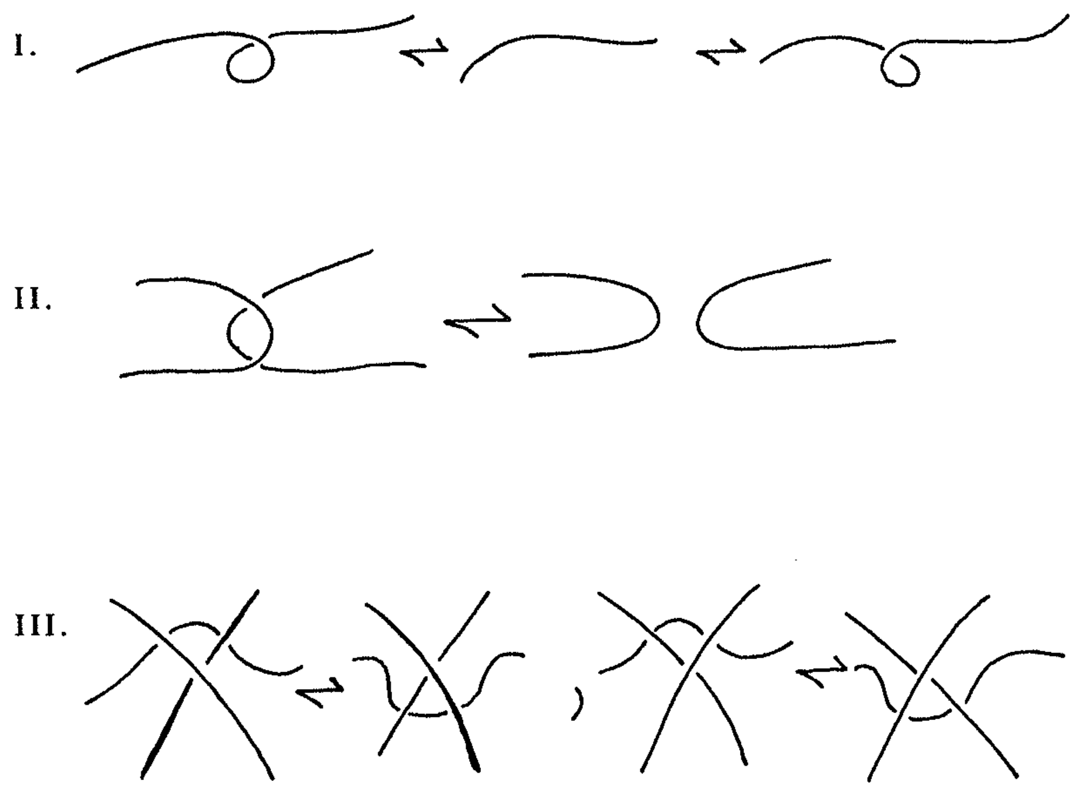


Knot Sets : Cancellation of Identicals

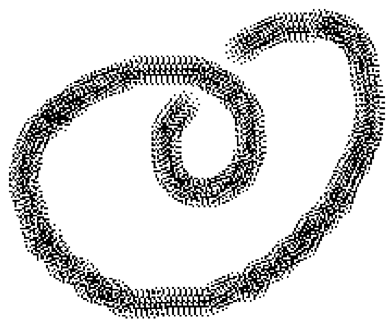


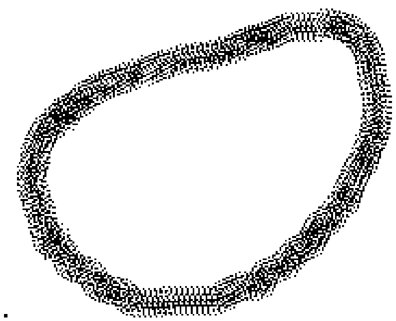
$$\{a, a\} = \{\ \}$$

Knot Sets are Invariant under Reidemeister Moves



Russell Paradox (K)not.

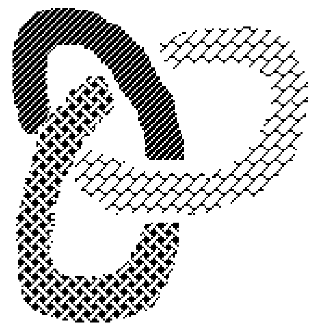




A belongs to A. ⟷ A does not belong to A.

Knots and Their Topology
Require
More
Structure

Three-Coloring a Knot Diagram



The Rules:

Either three colors at a crossing,

OR

one color at a crossing.

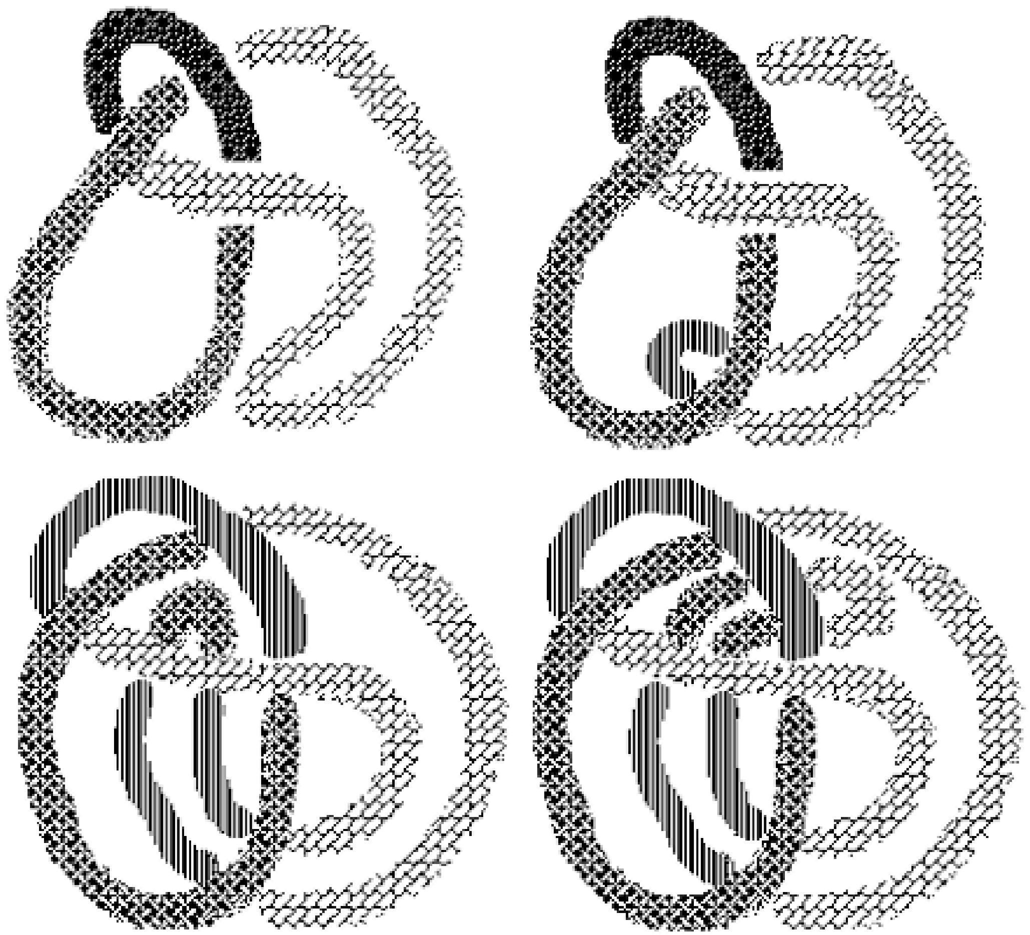


Figure 13 - Inheriting Coloring Under the Type Two Move

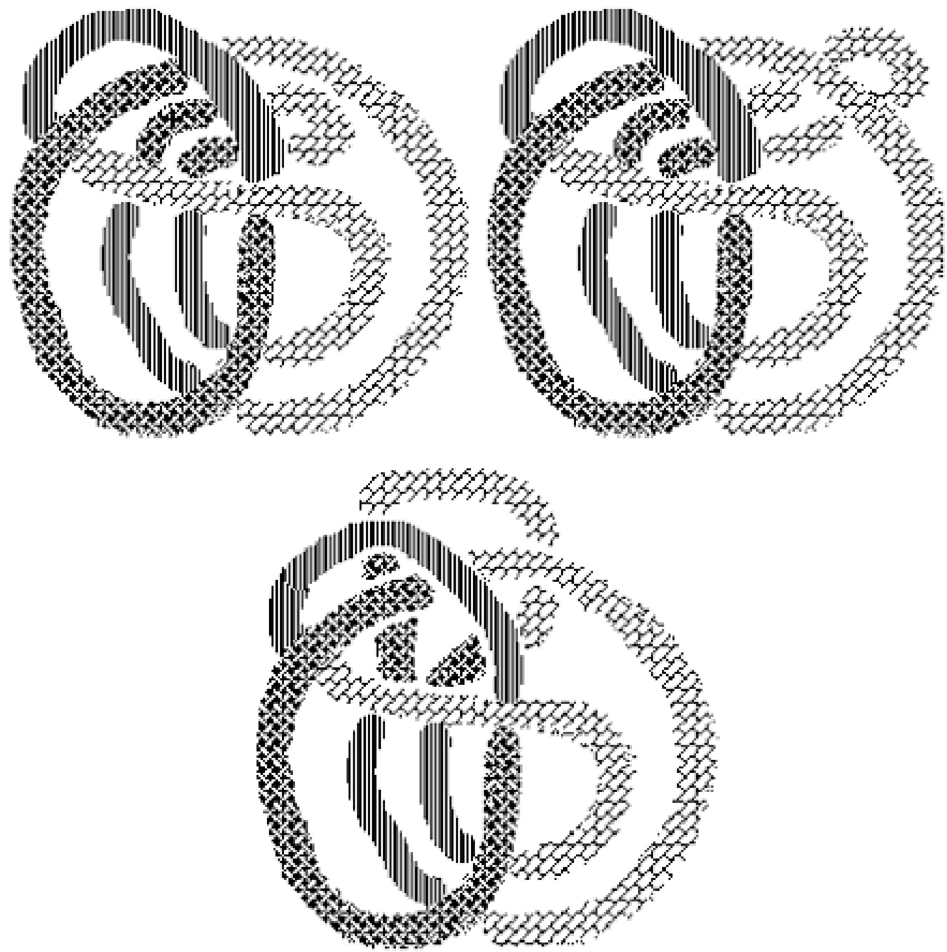
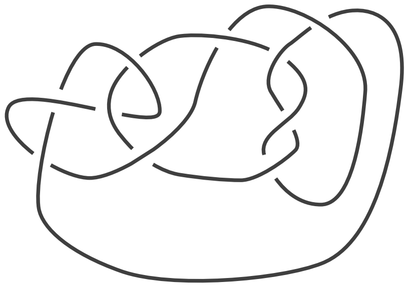
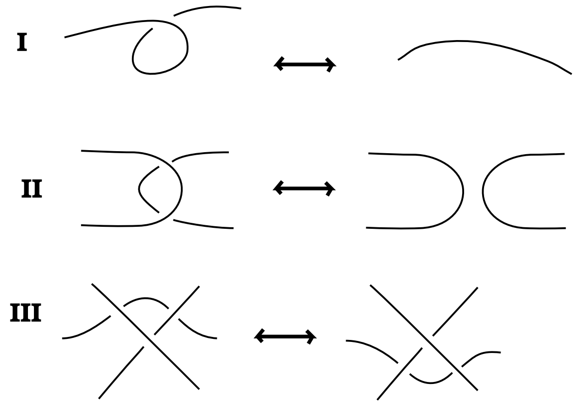


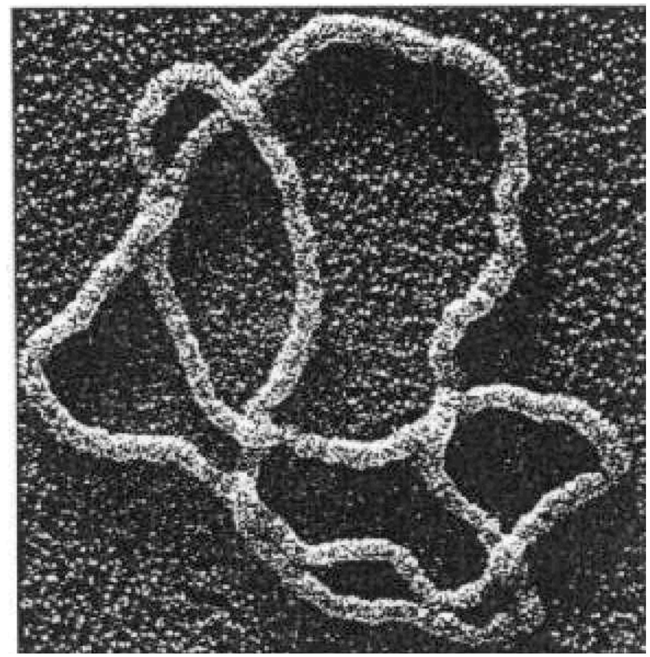
Figure 14 - Coloring Under Type Two and Three Moves

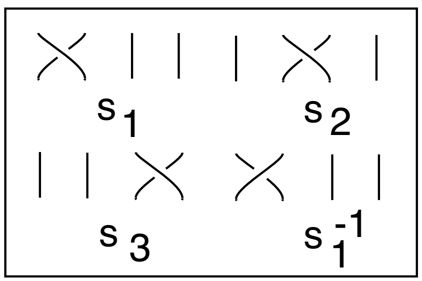
Via the three-coloring
we have proved that the
trefoil knot cannot be undone
using Reidemeister moves.
This is the simplest proof known
that the trefoil knot is non-trivial.

Knots, Links and Braids

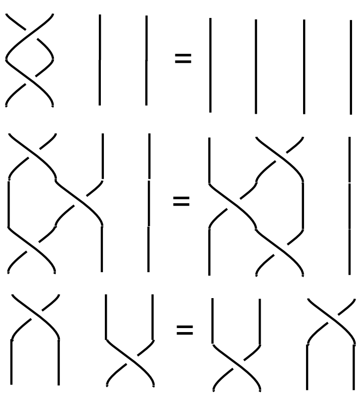








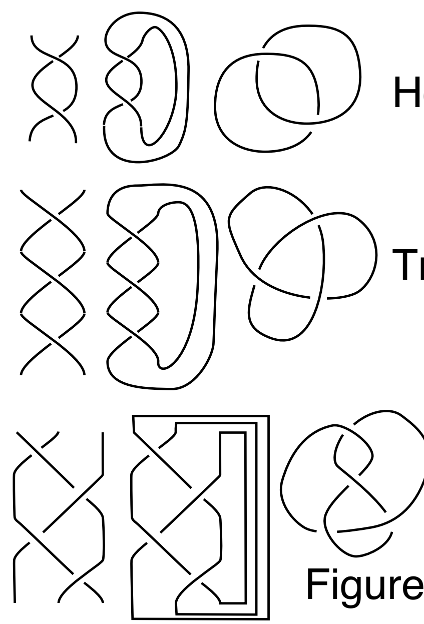
Braid Generators



$s_1^{-1} s_1 = 1$

$s_1 s_2 s_1 = s_2 s_1 s_2$

$s_1 s_3 = s_3 s_1$

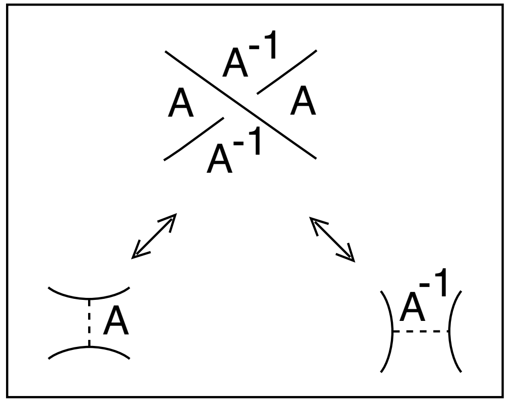


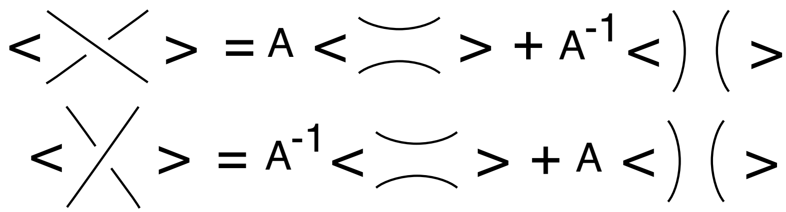
Hopf Link

Trefoil Knot

Figure Eight Knot

Bracket Polynomial Model for Jones Polynomial

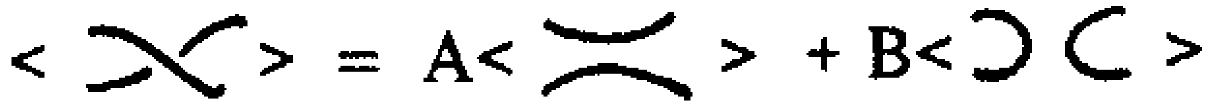




$$< K > = \sum_{S} < K|S > \delta^{||S||-1}.$$

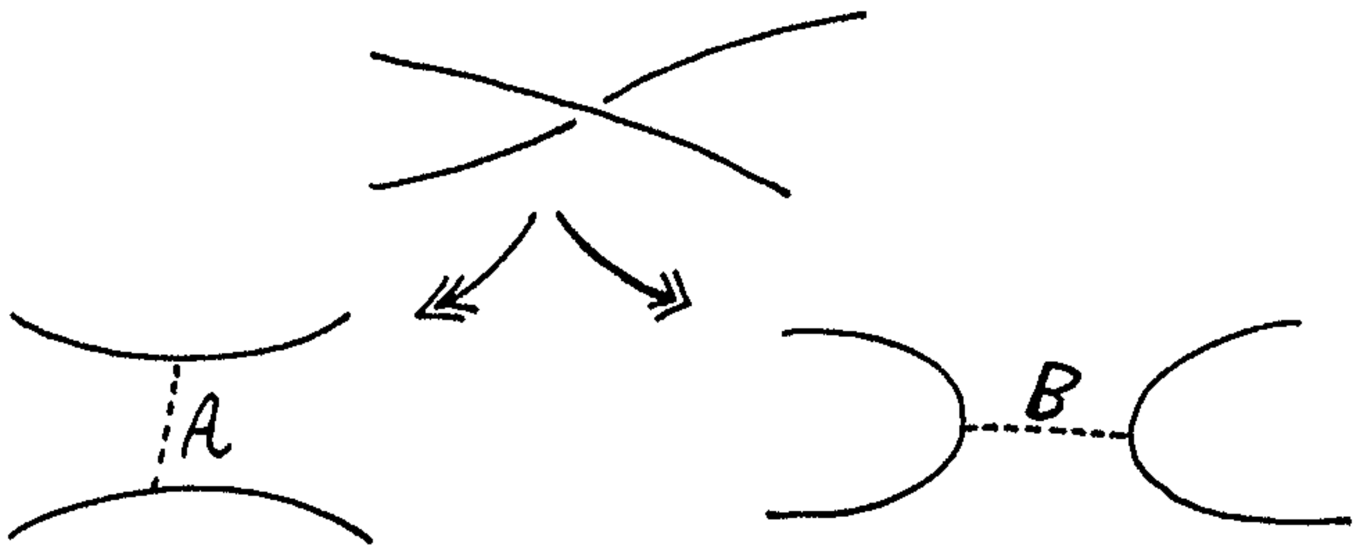
$$d = -A^{2} - A^{-2}$$

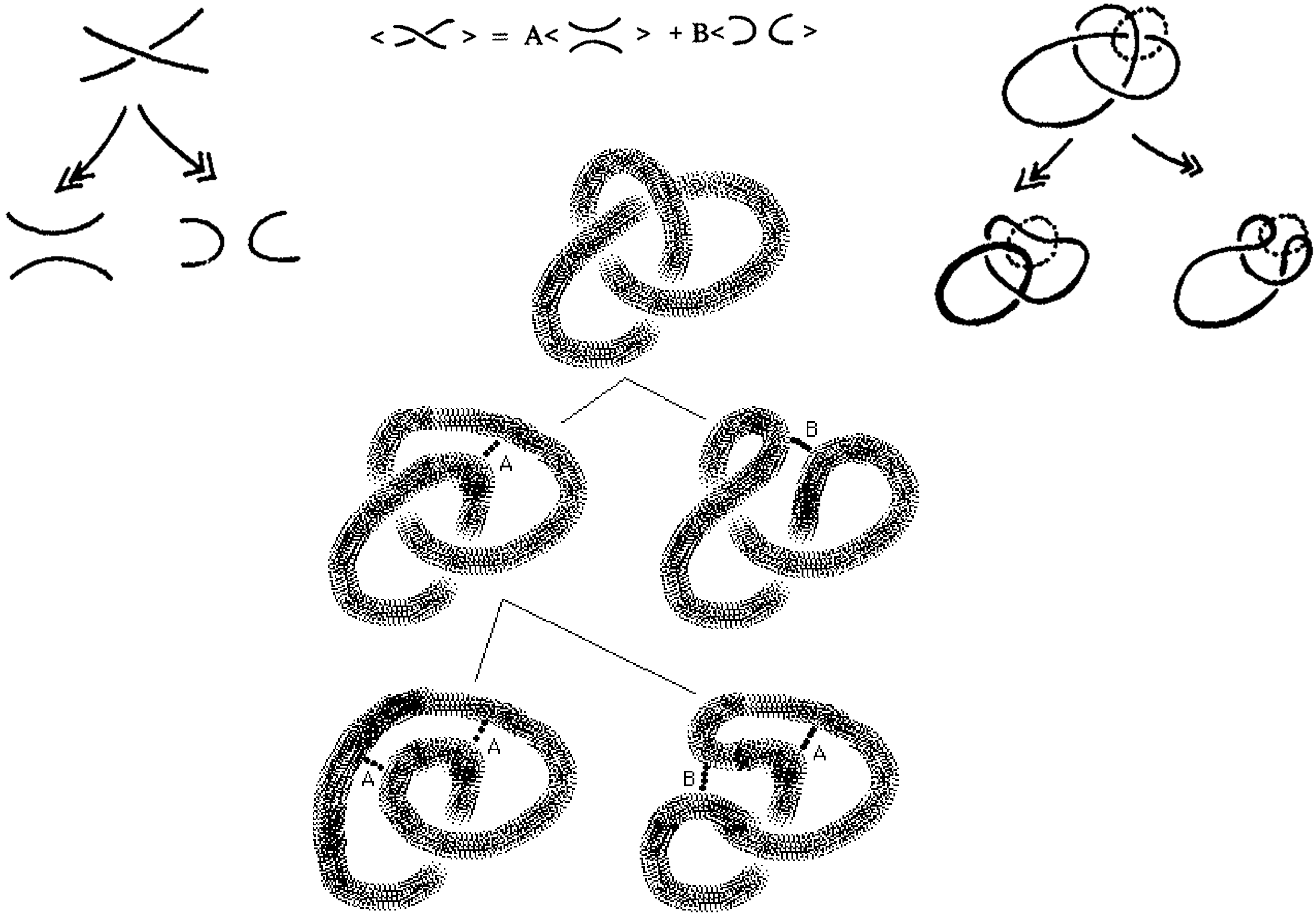
Exercise: Prove that the trefoil knot is topologically distinct from its mirror image.

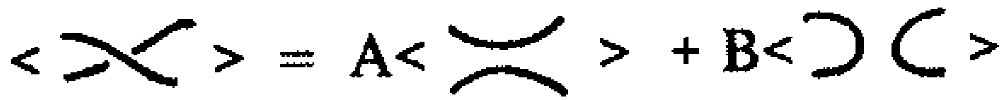


$$\langle O K \rangle = d \langle K \rangle$$

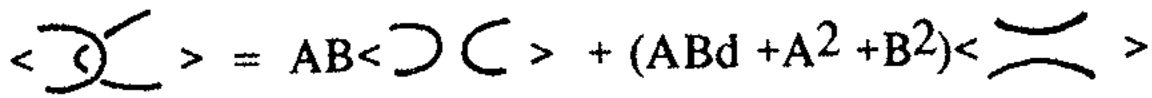
$$\langle O \rangle = 1.$$



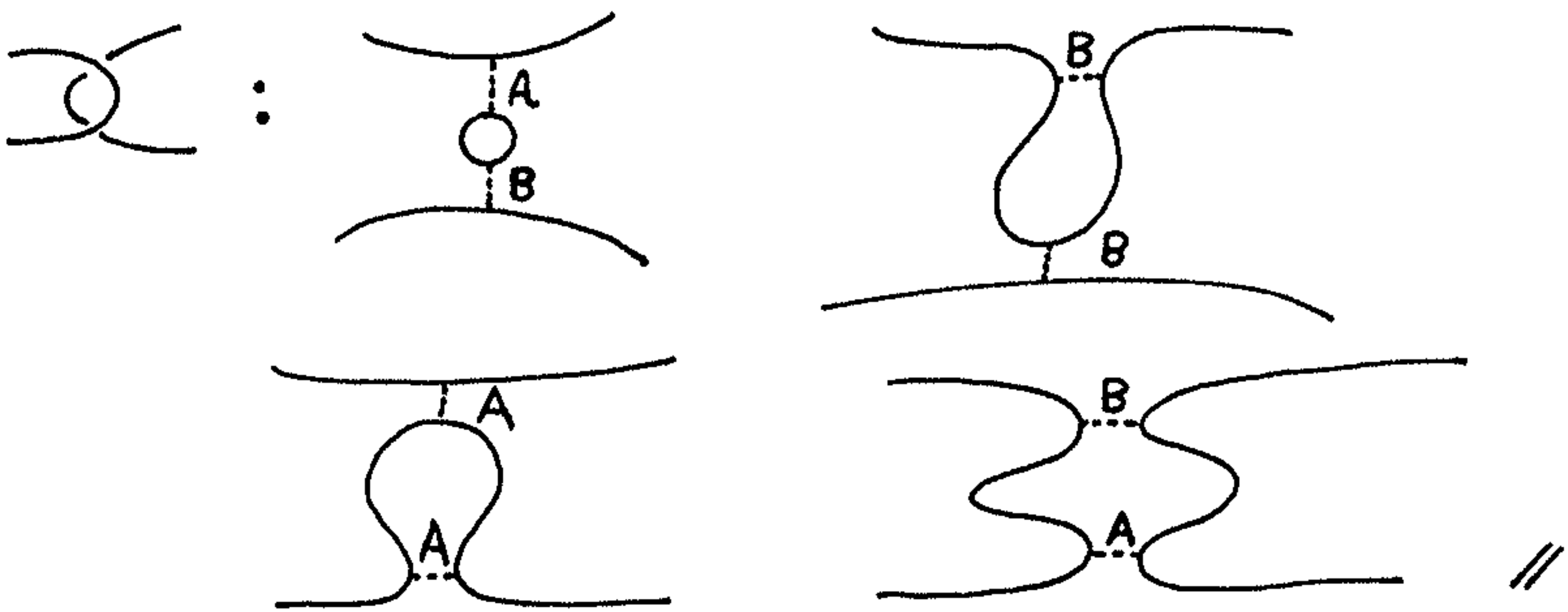

A
B
A
A
B
A



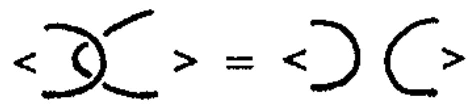
Lemma.



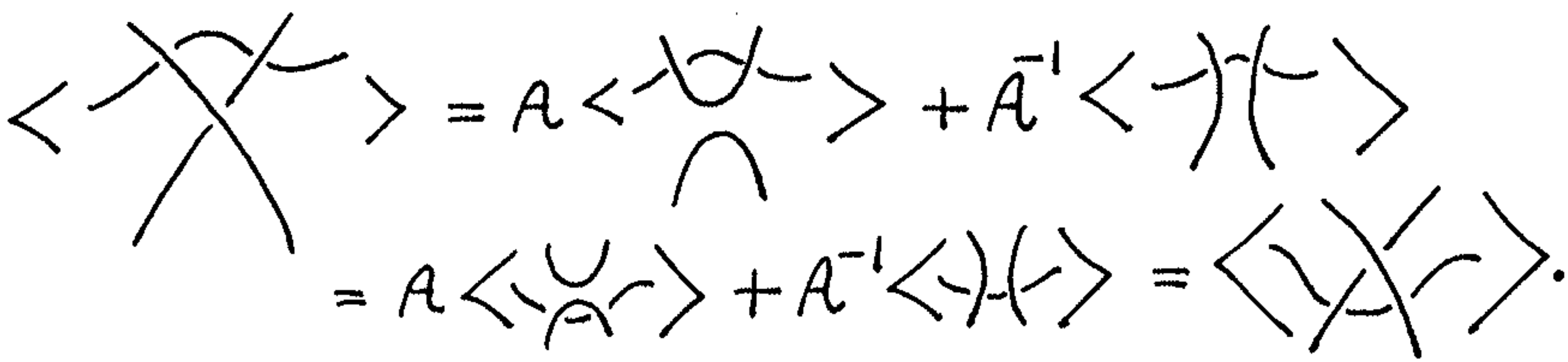
Proof.

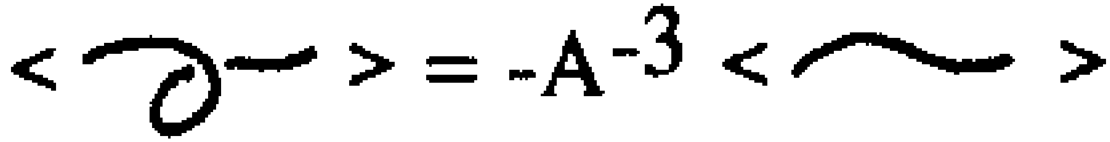


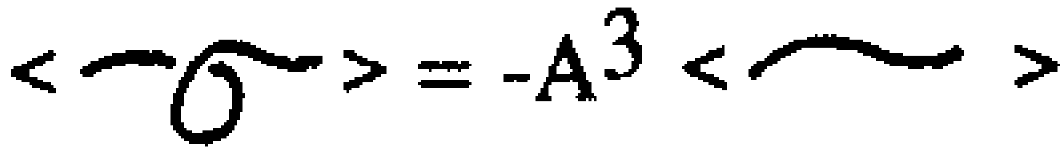
Hence we can achieve the invariance



by taking $B = A^{-1}$ and $d = -A^2 - A^{-2}$. A miracle happens, and we are granted invariance under the triangle move with no extra restrictions:





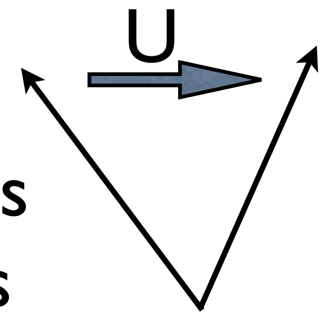


The Jones Polynomial $V_K(t)$.

$V_K(t) = f_K(t^{-1/4})$ where $f_K(A) = (-A^3)^{-w(K)}\langle K\rangle(A)$ where $w(K)$ is the sum of the crossing signs of the oriented link K, and <K> is the bracket polynomial obtained by ignoring the orientation of K.

Quantum Mechanics in a Nutshell

0. A state of a physical system corresponds to a unit vector |S> in a complex vector space.



1. (measurement free) Physical processes are modeled by unitary transformations applied to the state vector: |S> -----> U|S>

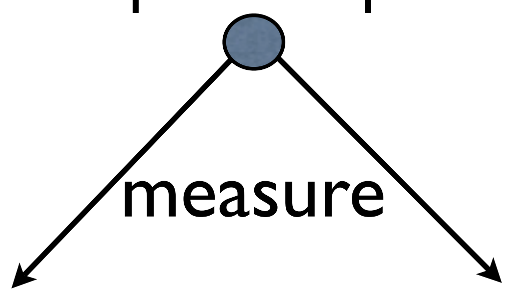
2. If |S> = z_1|1> + z_2|2> + ... + z_n |n>

in a measurement basis {|1>,|2>,...,|n>}, then measurement of |S> yields |i> with probability $|z_i|$^2.

Qubit

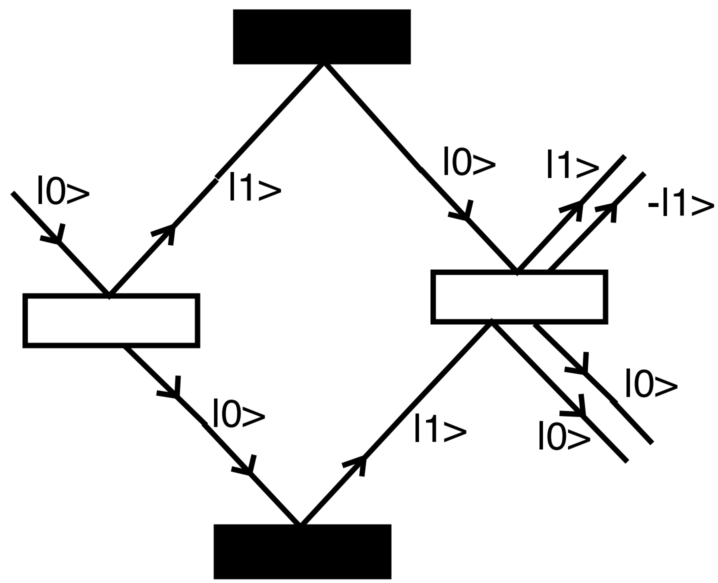
A qubit is the quantum version of a classical bit of information.

a|0> + b|1>

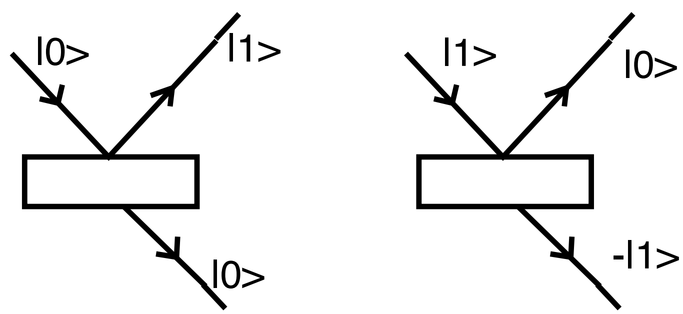


|0>
prob = |a|^2

|1>
prob = |b|^2



Mach-Zender Interferometer



$$H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} /\text{Sqrt}(2) \qquad M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

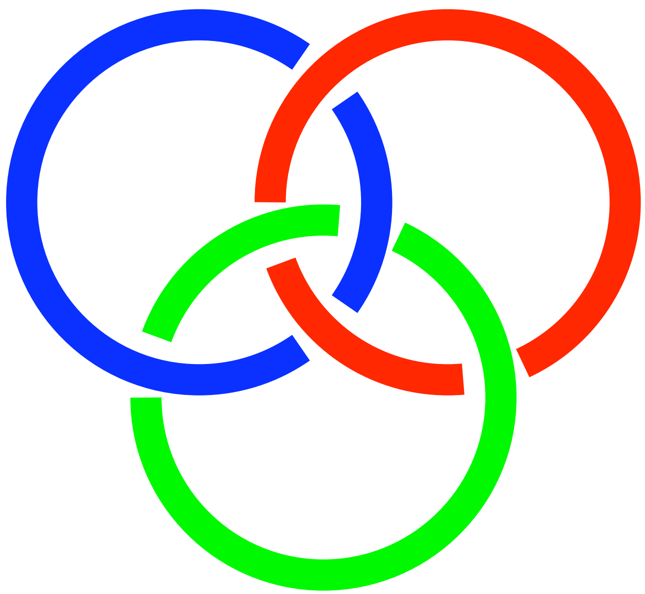
$$HMH = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Quantum Entanglement and Topological Entanglement

An example of Aravind [1] makes the possibility of such a connection even more tantalizing. Aravind compares the Borromean rings (see figure 2) and the GHZ state

$$|\psi\rangle = (|\beta_1\rangle|\beta_2\rangle|\beta_3\rangle - |\alpha_1\rangle|\alpha_2\rangle|\alpha_3\rangle)/\sqrt{2}.$$

(|000> - |111>)/Sqrt(2)



Is the Aravind analogy only superficial?!

Compare

|000>+|111>

and

|100>+|010>+|001>.

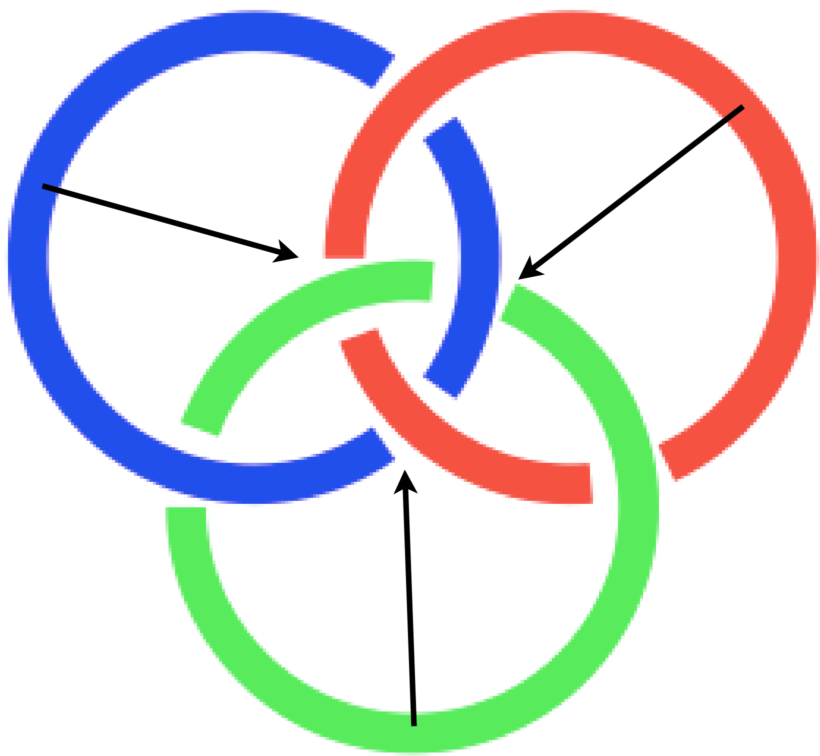
In the second case, observation in a given tensor factor yields an entangled state with 50-50 probability.

WHAT SORT OF LINK WOULD THAT BE?

In this way, we can make a case for quantum knots and links.

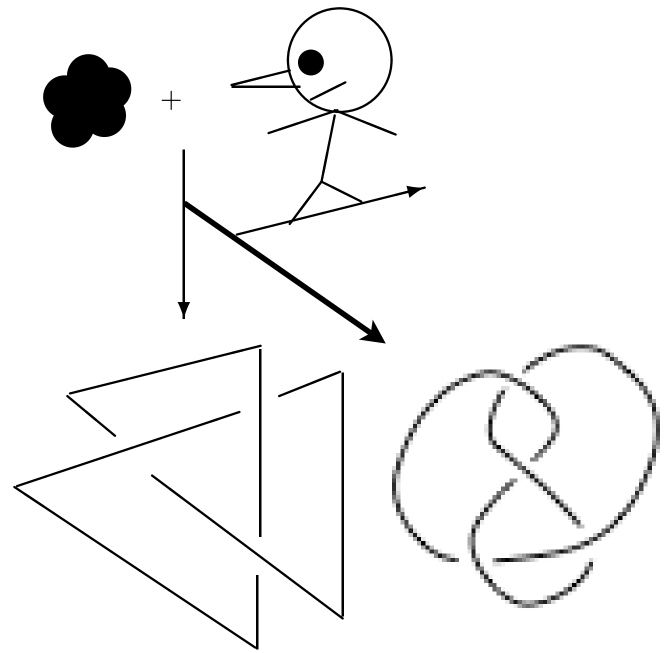
$$|100\rangle + |010\rangle + |001\rangle =$$
$$|1\rangle\{|0\rangle\} + |0\rangle\{|10\rangle + |01\rangle\}.$$

You can imagine a topological state that is a superposition of multiple link types.



Do we need Quantum Knots?

a|K> + b|K'>



K: probability |a|^2

K':probability |b|^2

K K'

Observing a Quantum Knot

Definition. A *quantum knot* is a linear superposition of classical knots.

(or a linear superposition of representatives for knot types.)

Knots, Gauge Fields and Quantum Gravity

Let $\psi(A)$ be a function of a gauge field A. Let

$$\hat{\psi}(K) = \int \mathcal{D}\mathcal{A}\psi(A)\mathcal{H}_K(A),$$

where the integral denotes your favorite notion of integrating over gauge fields (one chooses a heuristic, or fixes the gauge to allow a measure theory that can work) and $\mathcal{H}_K(A)$ denotes the trace of the holonomy of the gauge field taken around the specific embedding of the knot K in three dimensional space. This is the *loop transform* of the function $\psi(A)$ to a function $\hat{\psi}(K)$ of knotted loops in three dimensional space. The loop transform is not necessarily invariant under topological moves, but this is sometimes the case. We would like to, at least at the formal level, formulate an inverse transform to the loop transform. This would take the form

$$\check{\phi}(A) = \sum_{K \in \mathbf{K}} \phi(K)\mathcal{H}_K(A) = \phi(\sum_{K \in \mathbf{K}} \mathcal{H}_K(A))|K\rangle)$$

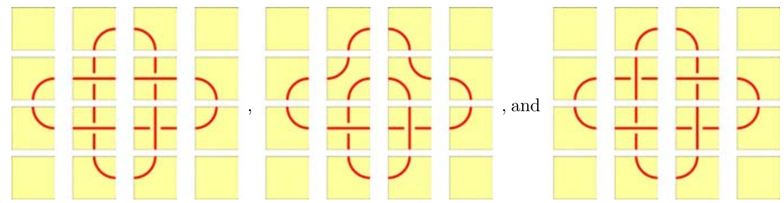
where $\phi(K)$ is a functional on knots and these sums would receive appropriate normalizations. Note that $\check{\phi}(A) = \phi(\mathcal{Q}_{\mathcal{H}}(\mathcal{A}))$ where $\mathcal{Q}_{\mathcal{H}}(\mathcal{A})$ is the quantum knot

$$\mathcal{Q}_{\mathcal{H}}(\mathcal{A}) = \sum_{K \in \mathbf{K}} \mathcal{H}_K(A))|K\rangle.$$

While it is impractical to consider integrating over all possible embeddings of a circle into three dimensional space, it is mathematically possible to examine summations involving all knot types. In this way the notion of quantum knot is inextricably tied to these questions about the loop transform. The loop transform is of particular value in the quantum gravity theory of Ashtekar, Smolin and Rovelli.[31]

Quantum knots and mosaics

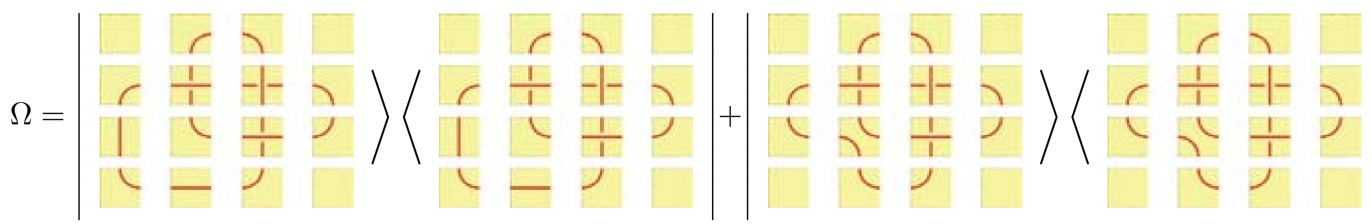
with Sam Lomonaco and the subject of the NEXT TALK!



Each of these knot mosaics is a string made up of the following 11 symbols

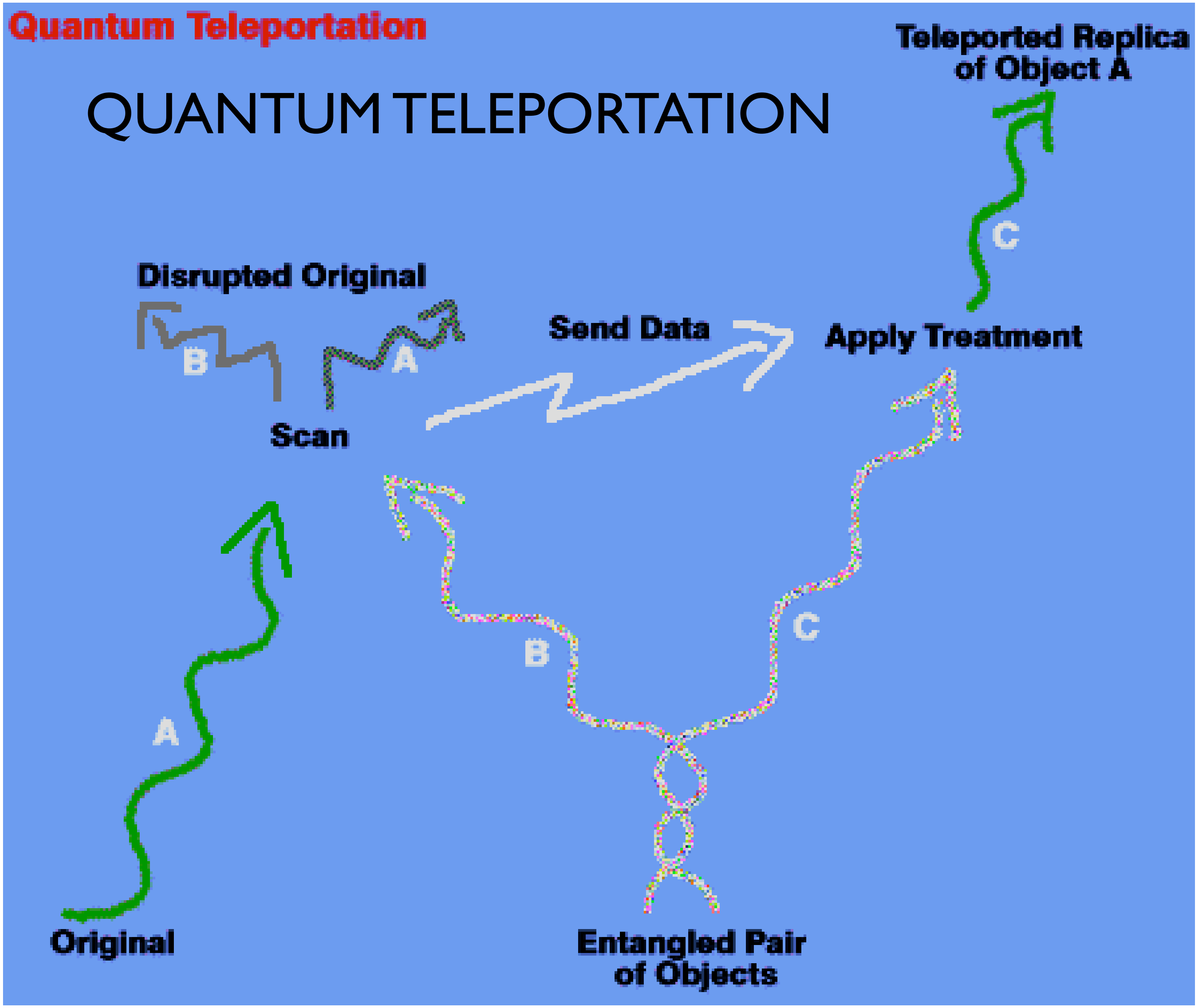
called *mosaic tiles*.

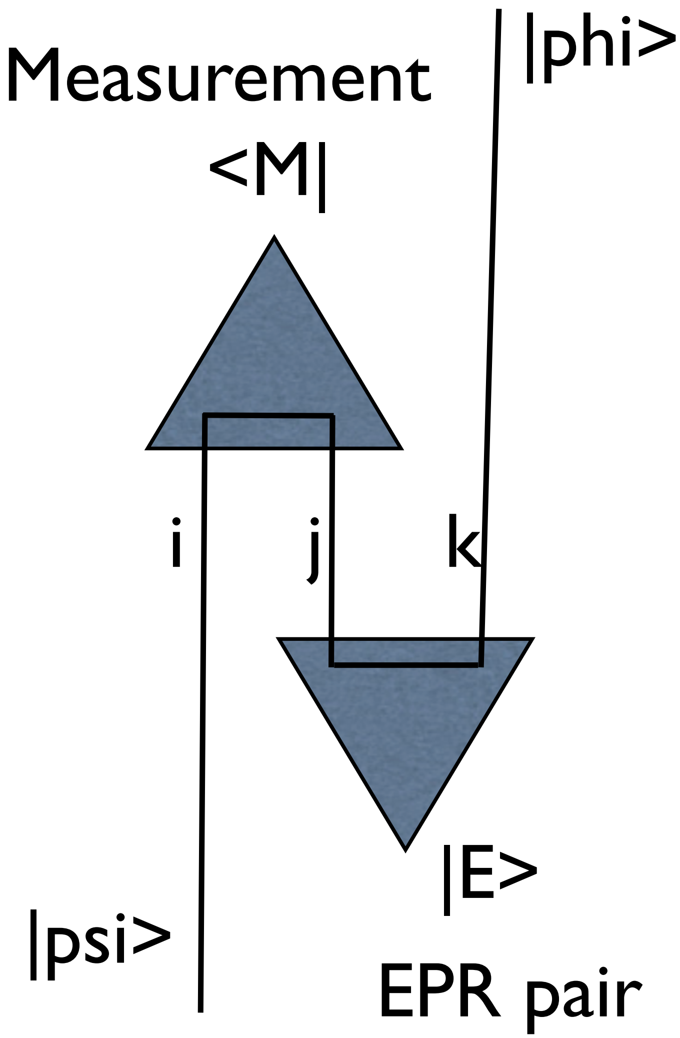
Each mosaic is a tensor product of elementary tiles.



This observable is a quantum knot invariant for 4x4 tile space. Knots have characteristic invariants in nxn tile space.

QUANTUM TELEPORTATION





$$\langle M| = \sum M_{ij} \langle ij|$$

$$|E\rangle = \sum E_{ij} |ij\rangle$$

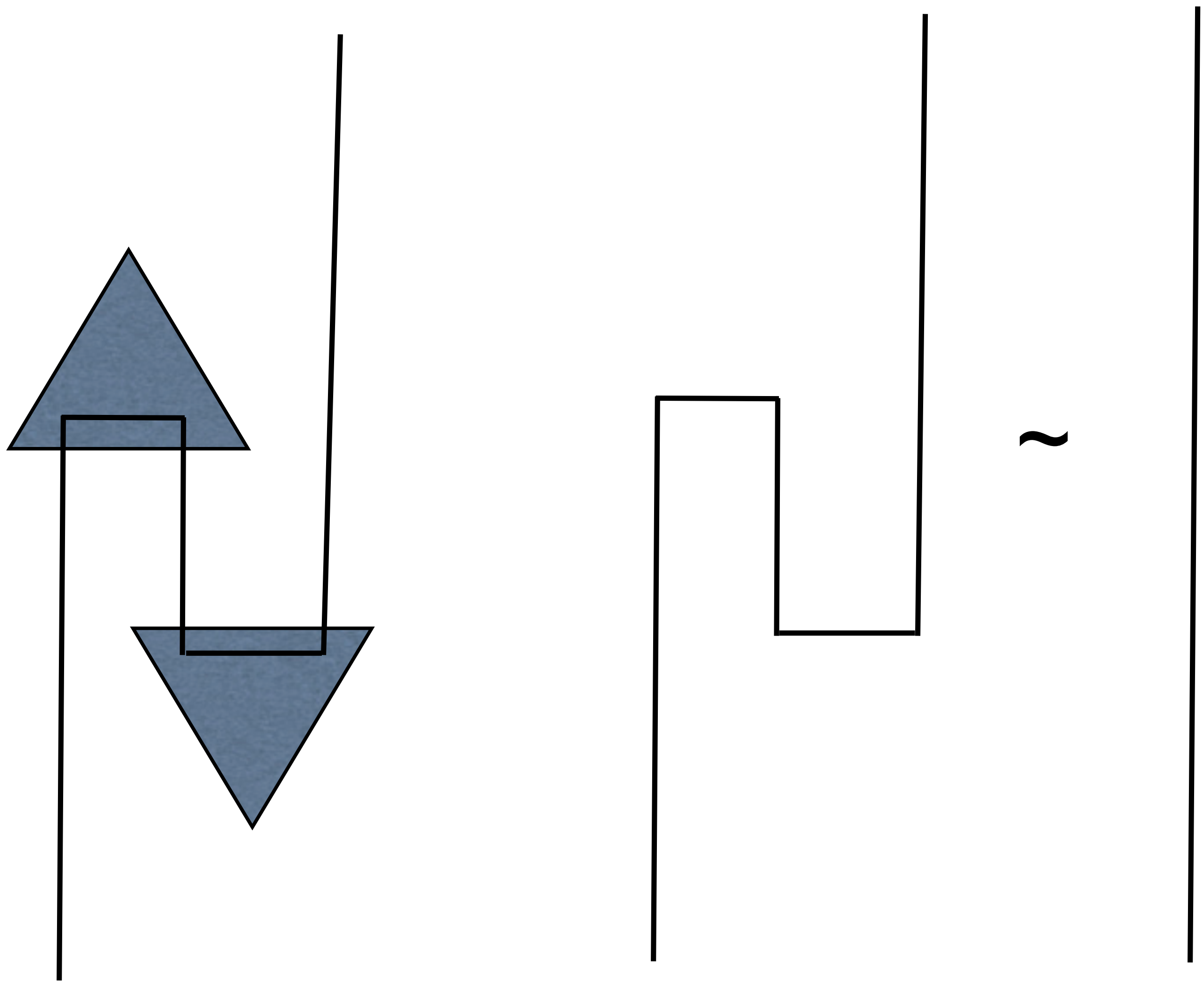
$$|\phi\rangle_k = \sum |\psi\rangle_i M_{ij} E_{jk}$$

$$|\phi\rangle = (ME)^t |\psi\rangle$$

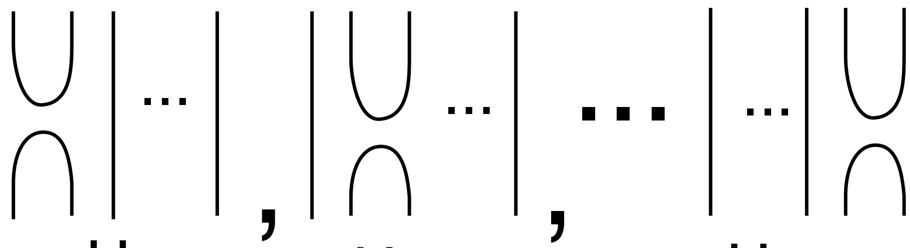
When ME = Identity,
then $|\phi\rangle = |\psi\rangle$.

Teleportation is achieved
by choosing an orthonormal
meaurement basis where one member
inverts E, and the other members
are unitary rotations
away from the key inverting member.

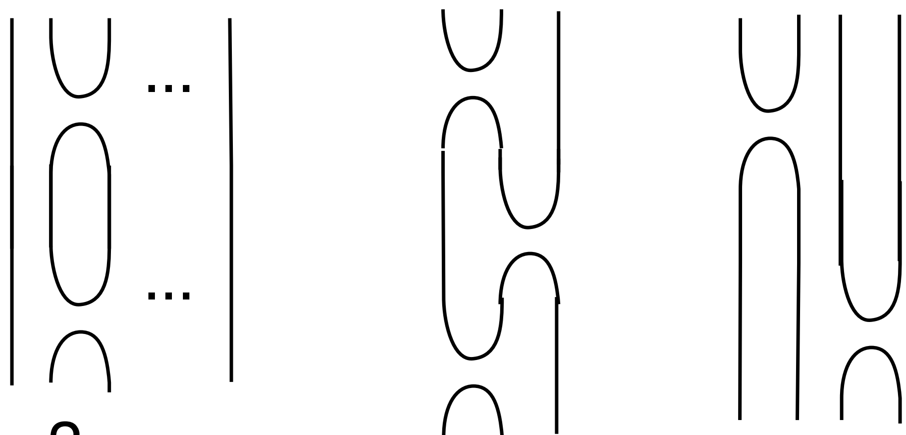
Teleportation Topology - Bare Bones



Relations in the Temperley-Lieb Algebra



U_1 , U_2 , ... U_{n-1}



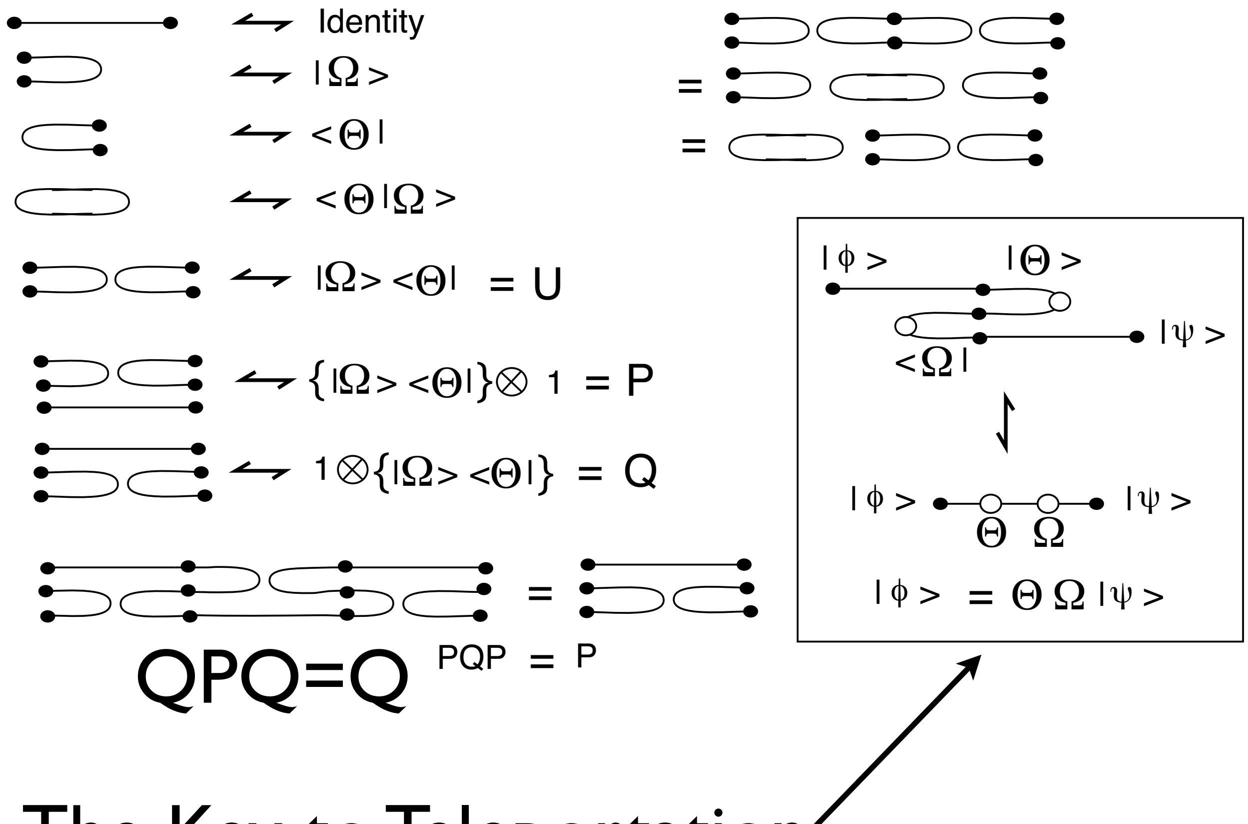
$U_i^2 = \delta U_i$

$U_i U_{i+1} U_i = U_i$

$U_i U_j = U_j U_i$

if $|i - j| > 1$.

The Temperley-Lieb Category



The Key to Teleportation

Diagrammatics
Logic
Topology
Categories

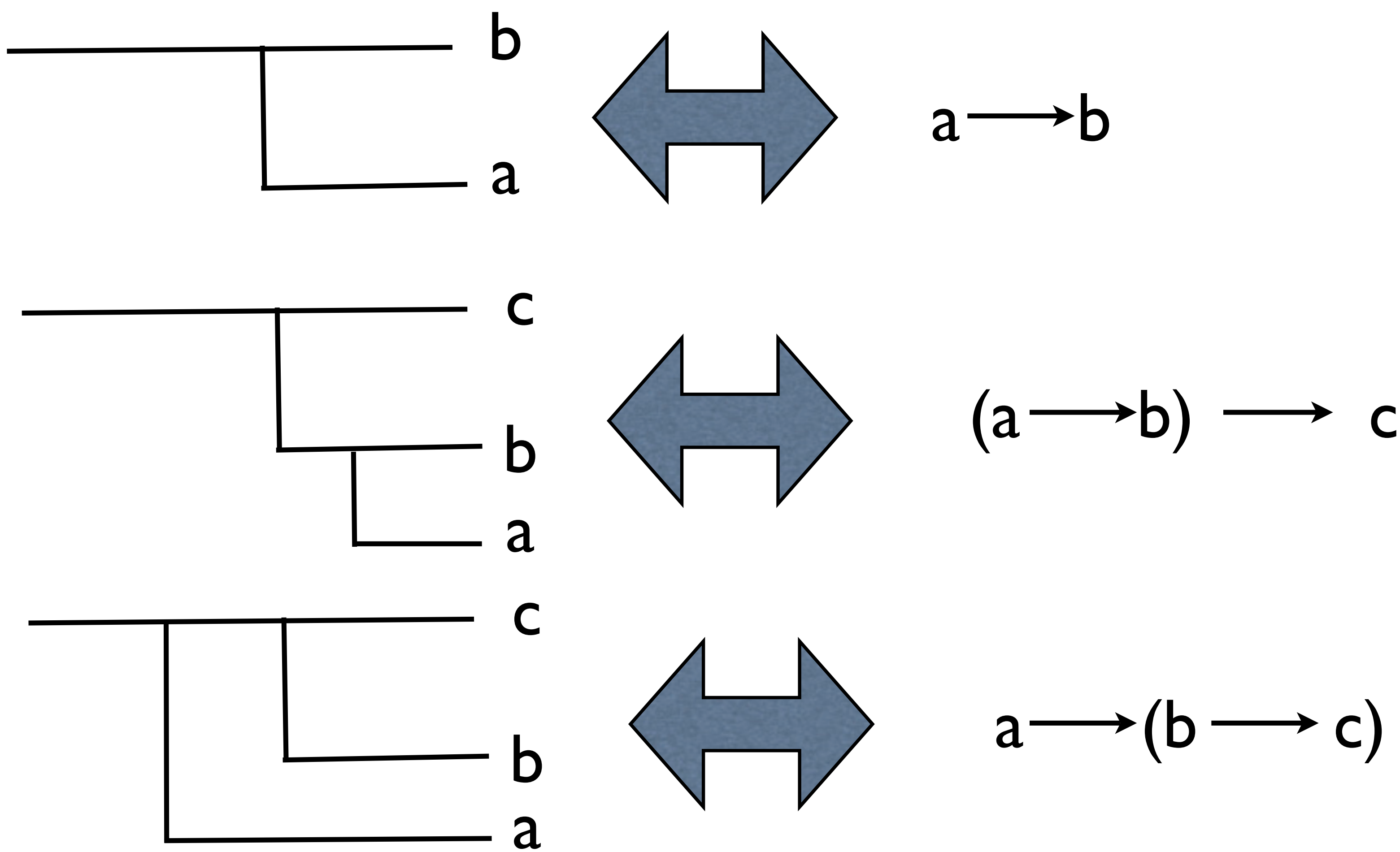
PreGeometry as a Calculus of Propositions?

We go back to Gottlob Frege and
Charles Sanders Pierce in search
of a structure deeper than
Boolean algebra.

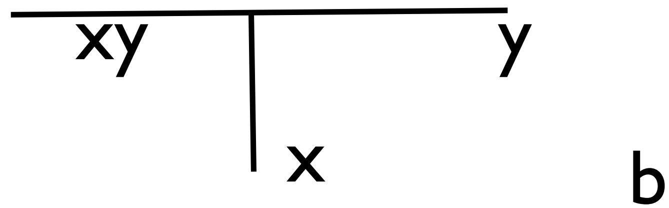
Frege’s Begriffsschrift -- Conceptual Notation

1879

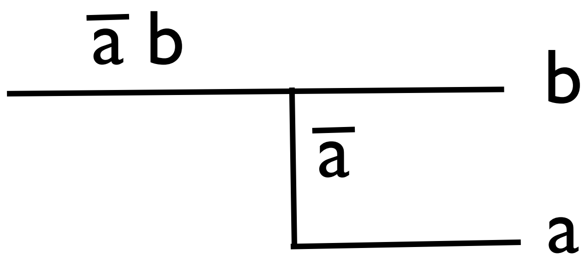
Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens

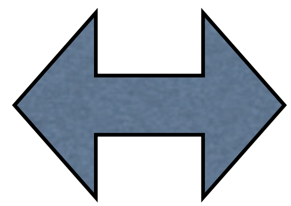


Frege's Conceptual Notation Decoded



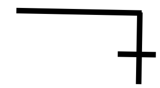
Negation is a 90 degree bend.





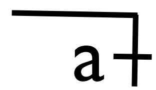
$(a \longrightarrow b) = \bar{a}\, b$

Non-Associative
Operations
Indicated Through a
Category of Trees

C. S. Peirce's Sign of Illation 

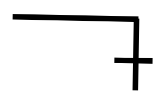
Now write a + b for a OR b.

$$(a \longrightarrow b) = \bar{a} + b$$

Peirce wrote 

$b = \bar{a} + b$

creating the pormanteau
sign of illation



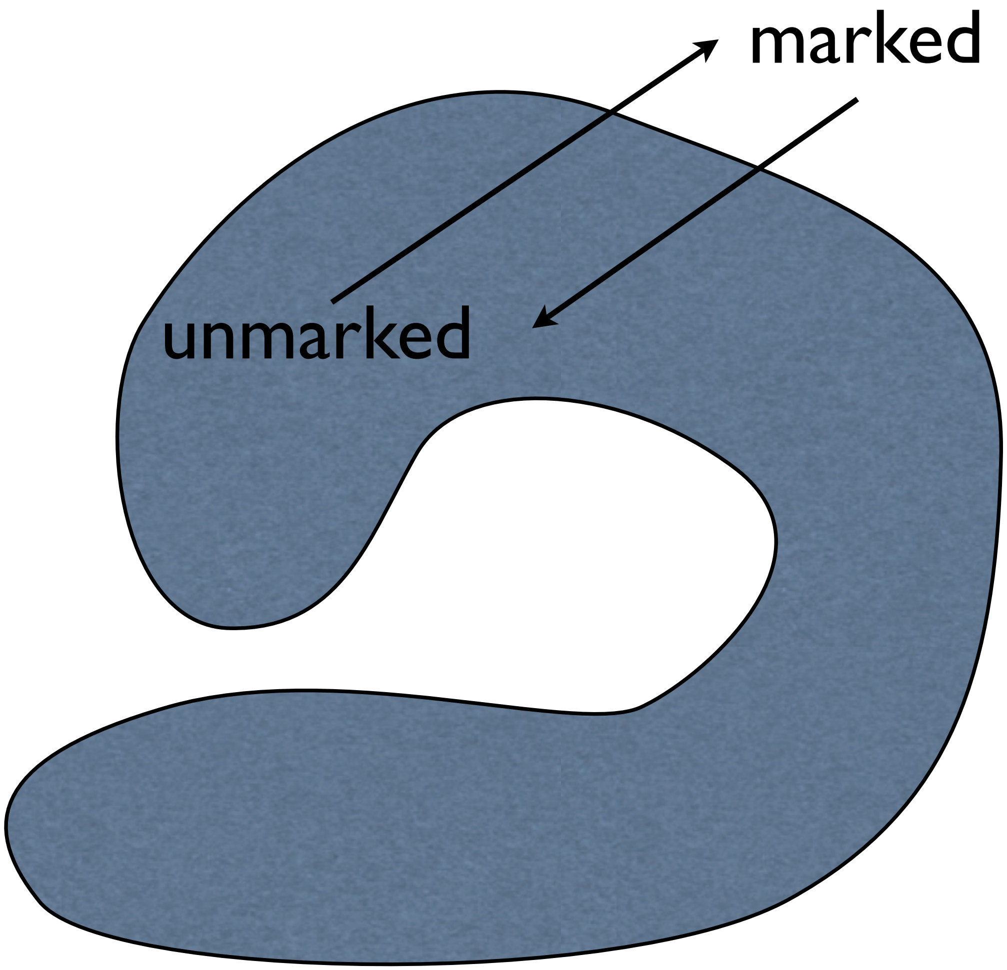
⌉ ⌉+

G. Spencer-Brown in
"Laws of Form"
used essentially the Peirce sign of illation
but writes

⌉ instead of ⌉+

and uses ab for
a OR b.

In Spencer-Brown, the mark ⌉ is regarded
as indicating either the act of crossing
the boundary of a distinction, or as the
"marked state" of a distinction.



⏋X⏋ = state obtained by crossing from X.

⏋unmarked⏋ = marked

⏋ ⏋ = ⏋

⏋marked⏋ = unmarked

⏋⏋ =

In Spencer-Brown there is a single "logical particle", the mark ⏋ .

A single "logical particle",
the mark $\urcorner$.

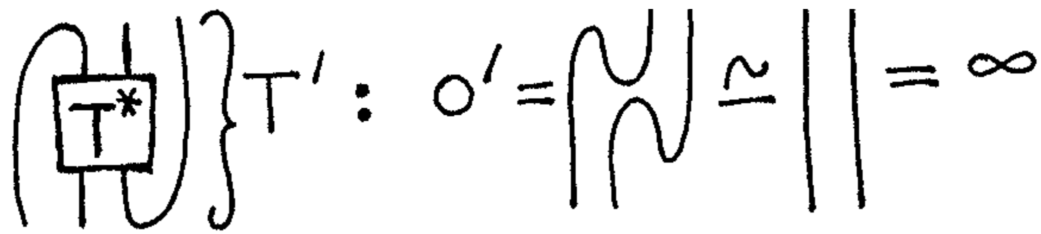
The mark interacts with itself in two ways, either producing nothing, or producing itself.

$$\urcorner\!\!\urcorner \; = \qquad$$

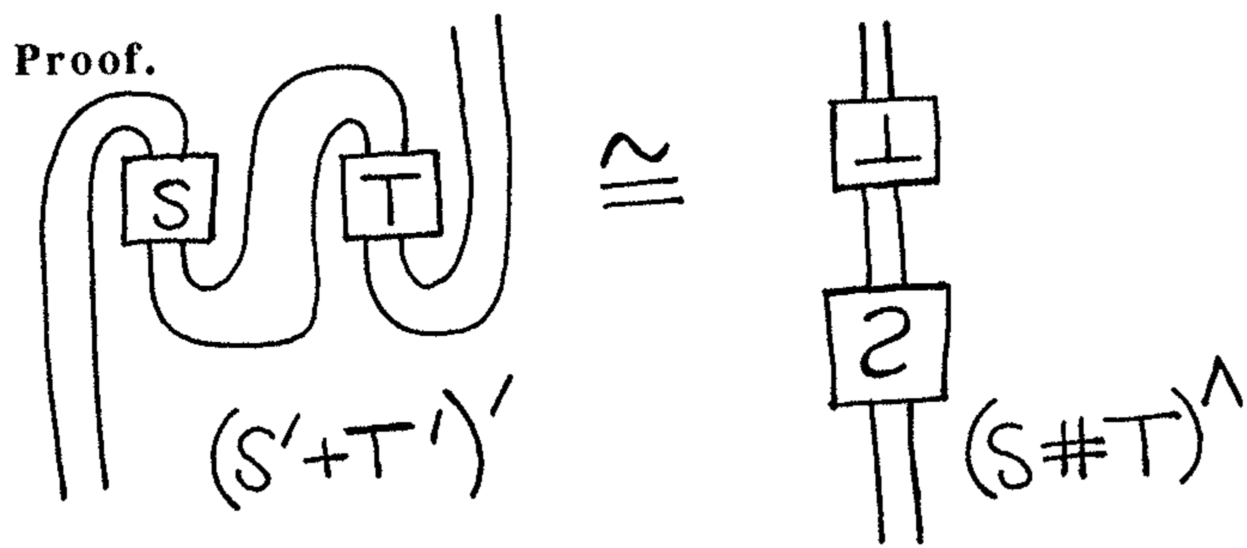
$$\urcorner\;\urcorner \; = \; \urcorner$$

Digression: What is going on about 90 degrees and negation?

Compare with tangle theory where a 90 degree turn applied to x yields -1/x.

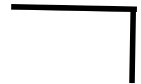


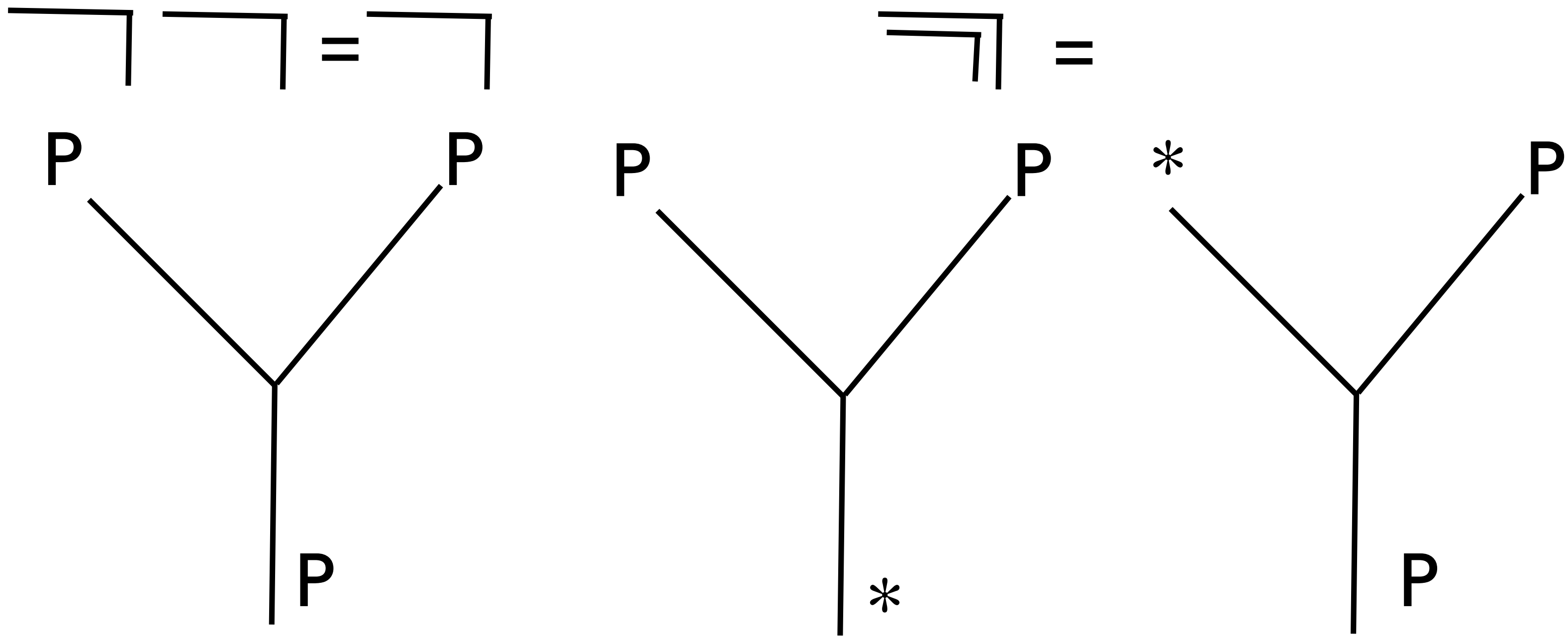
And DeMorgan's Law lives in a topological category.



The Logical Particle is a Fibonacci Particle

* = unmarked state

P = marked state = 



Letting PP denote P or * we write symbolically

$$PP = P + *.$$

And P becomes the Golden Mean.

Interactions of P with itself generate Fibonacci numbers.

P^2 = P + *

P^3 = PP + *P = P + * + P = 2P + *

P^4 = 3P + 2*

P^5 = 5P + 3*

P^6 = 8P + 3*

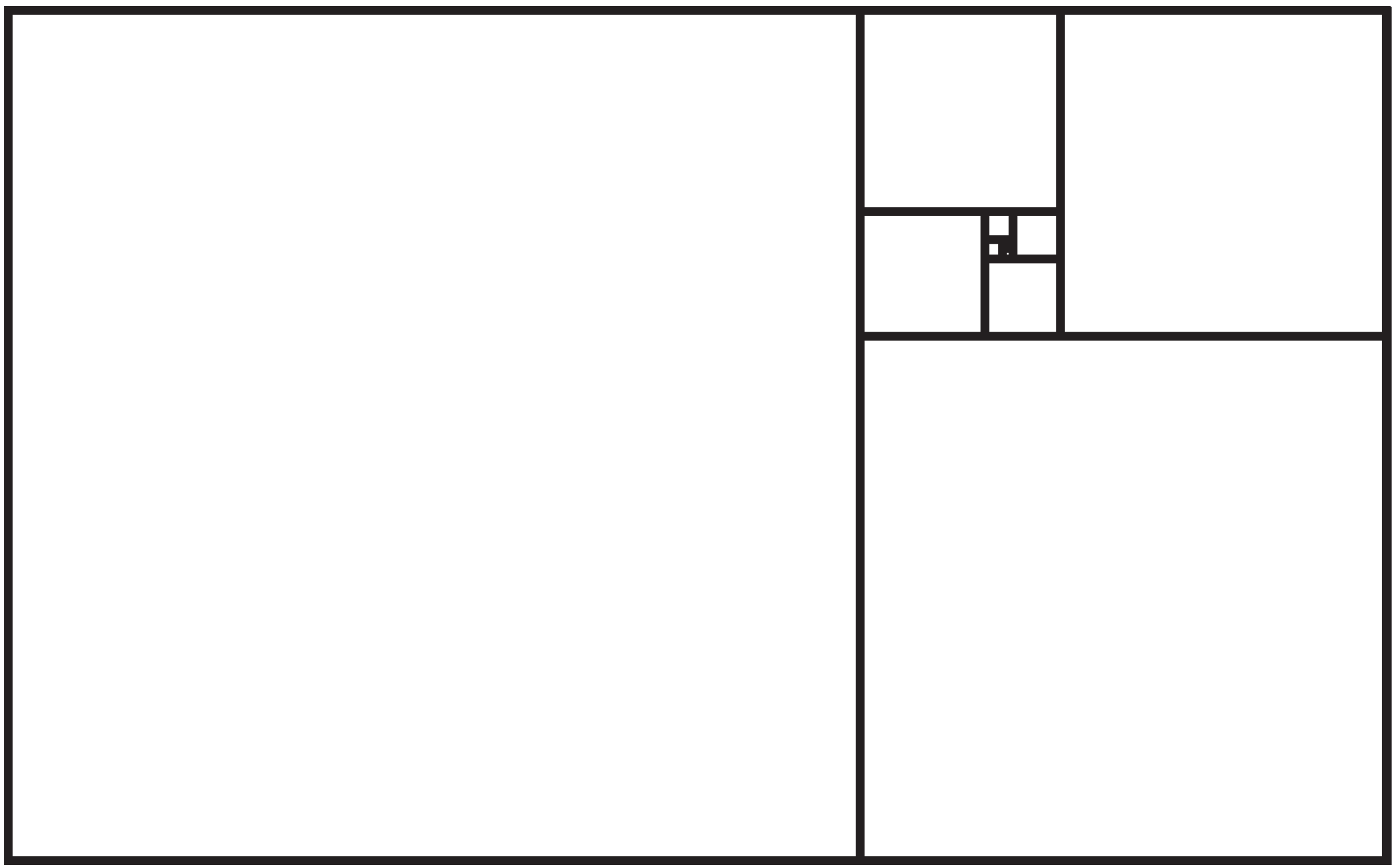
P^7 = 13P + 8*

The Golden Rectangle:

$$PP = P + 1$$

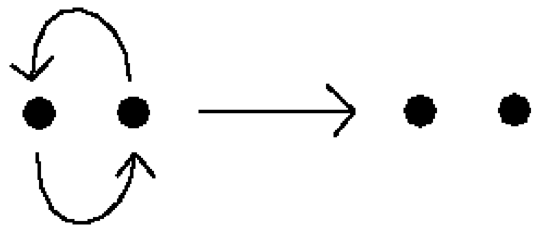
$$P = 1 + 1/P$$

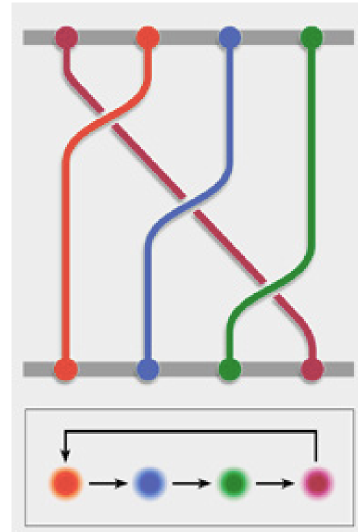
$$1/P = (P-1)/1$$

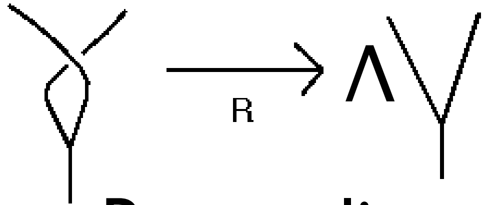


Remarkably, this primitive Fibonacci particle
takes part in a braided tensor category
that generates a unitary representation of
the Artin braid group that is dense in
the unitary groups.
This representation can be used for
universal topological quantum computation
and for studying quantum algorithms that
compute Jones polynomials.

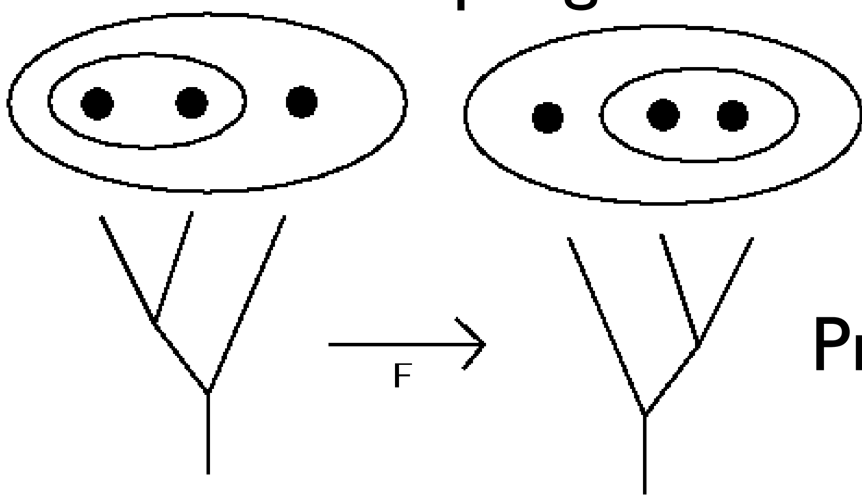
Braiding Anyons





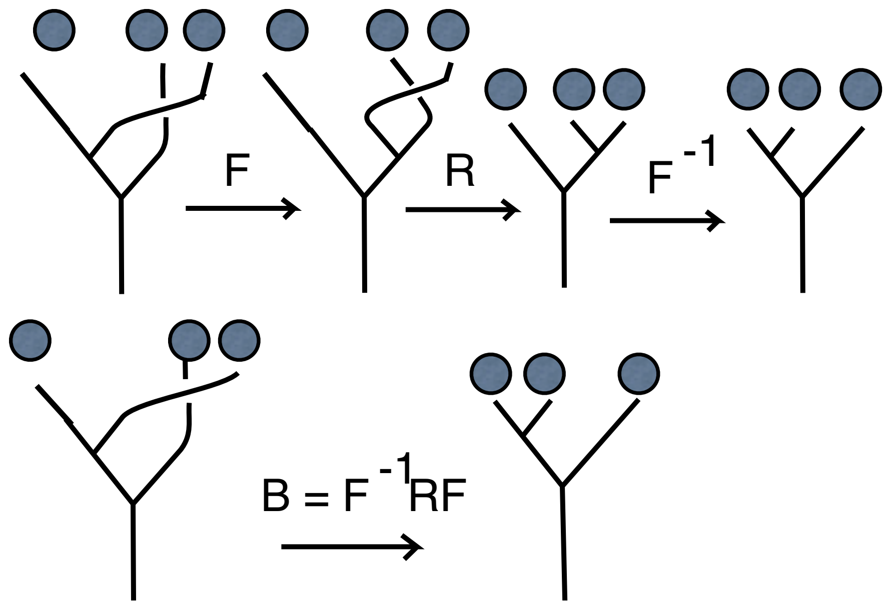


Recoupling

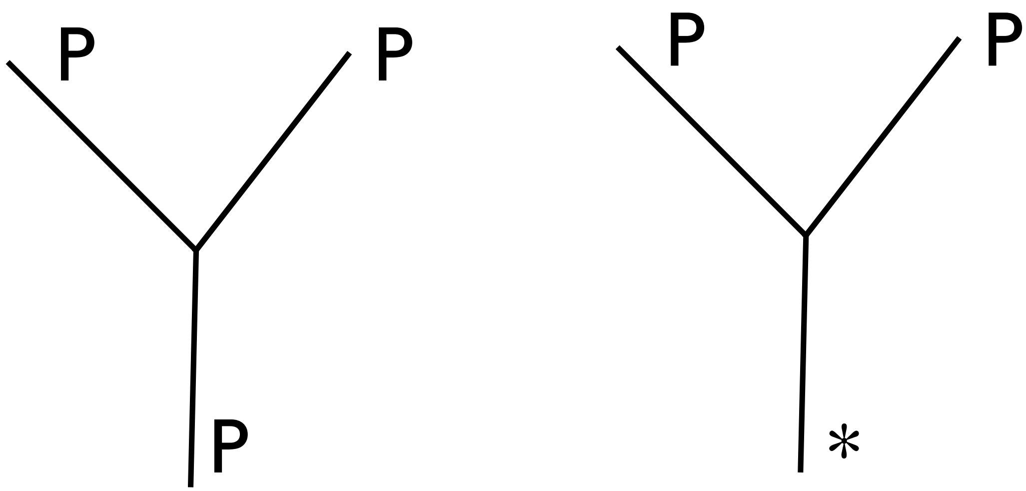


Process Spaces

Non-Local Braiding is Induced via Recoupling



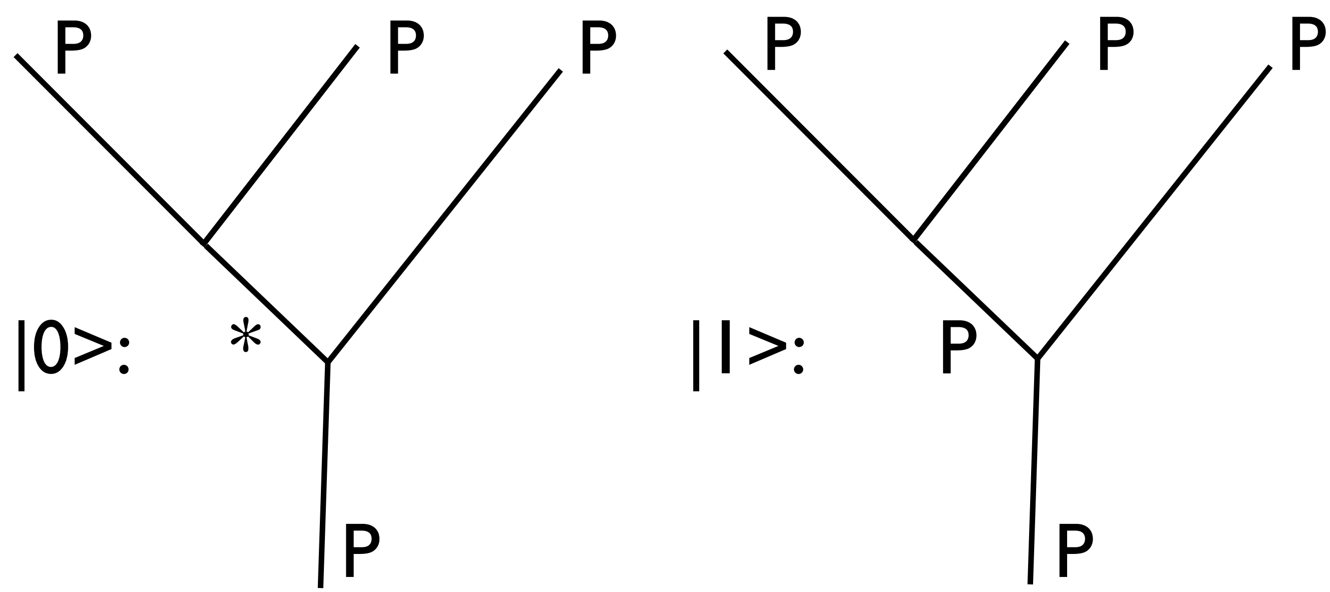
Fibonacci Process

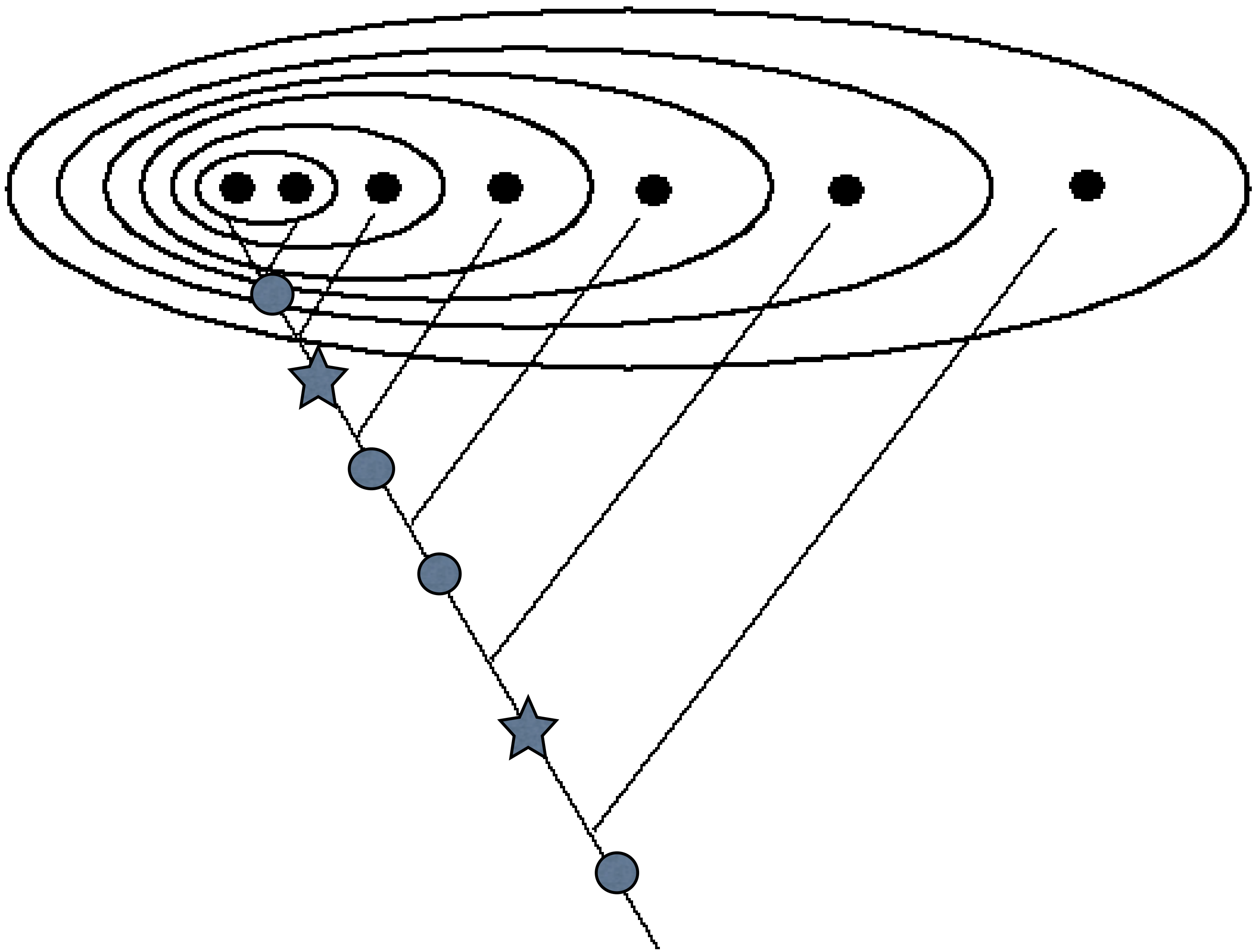


The "particle" P interacts with P
to produce either P or *.
The particle * is neutral.

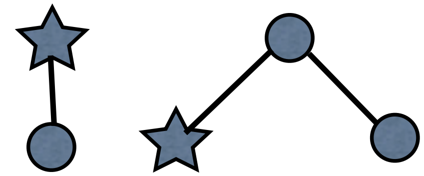
THE THREE STRAND BRAID GROUP CAN ACT ON A SINGLE QUBIT SPACE.

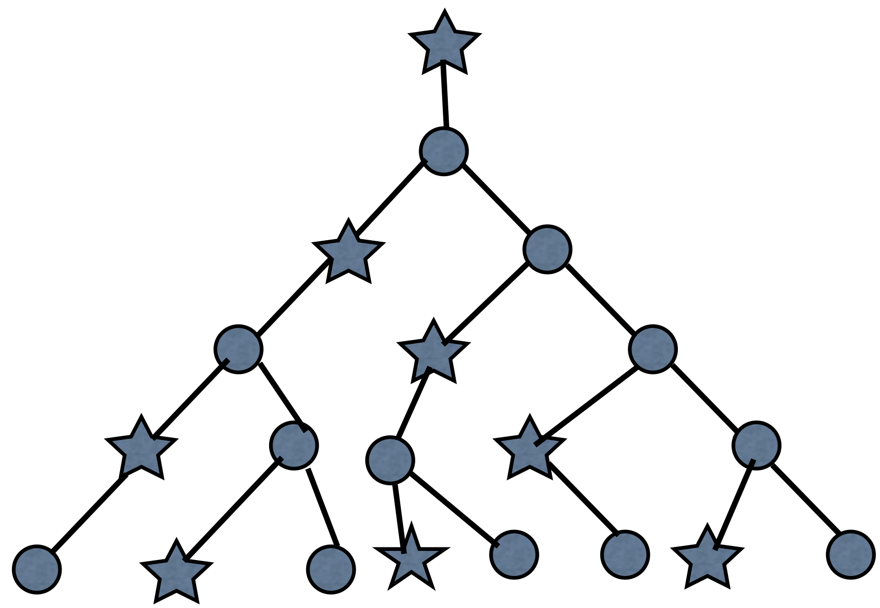
The process space with three input P's and one output P has dimension two.
It is a candidate for a unitary representation of the three strand braids.





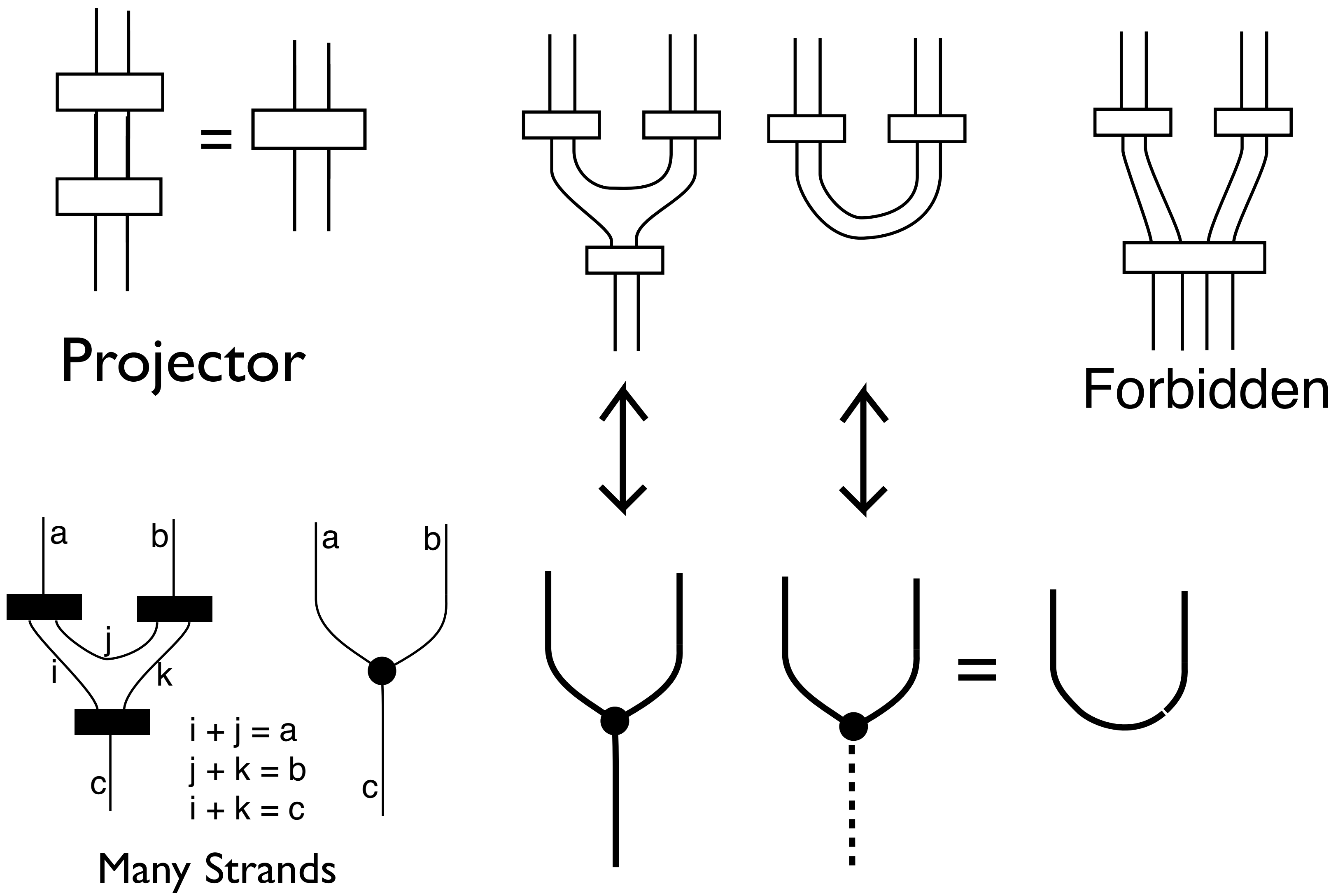
Fibonacci Tree:

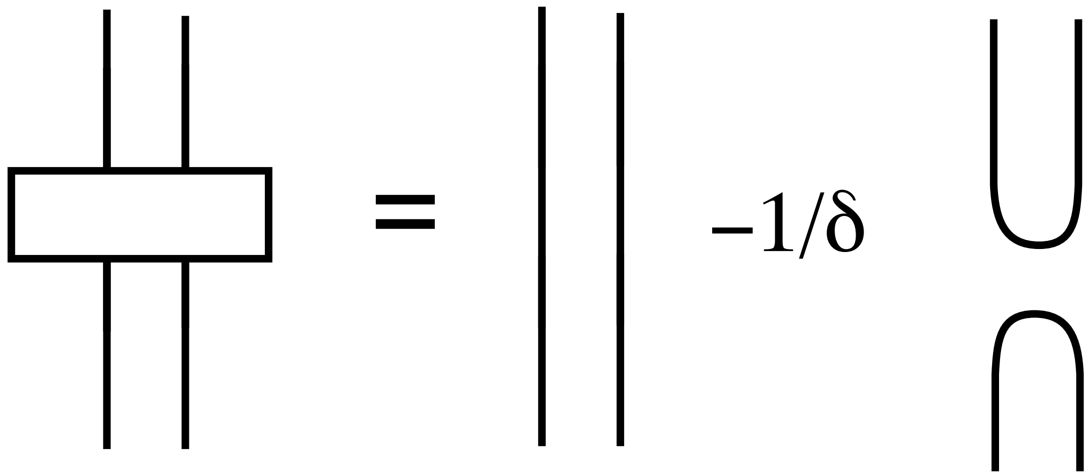


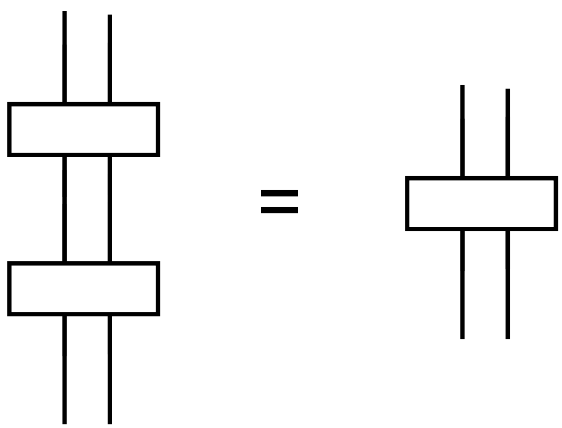


Admissible Sequences
are the Paths from the Root

Double Stranded Iconics for Fibonacci Model




=
−1/δ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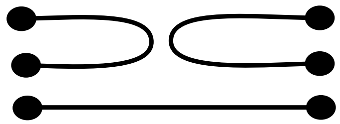
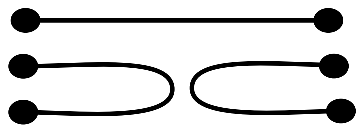
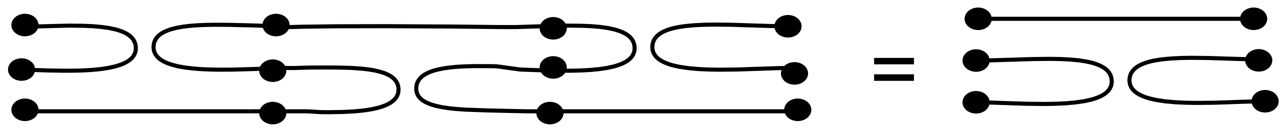
Topology and Temperley-Lieb



P



Q



PQP = P

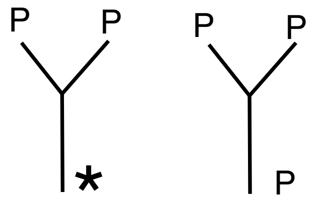
P = > <   PP = > <> < = <> > = <> P

Q = } {

PQP = > < } { >< = <}{> > < = <}{> P

Temperley-Lieb Relations Implicit in Projector Structure

Fibonacci Model



$$\delta = -A^2 - A^{-2}$$

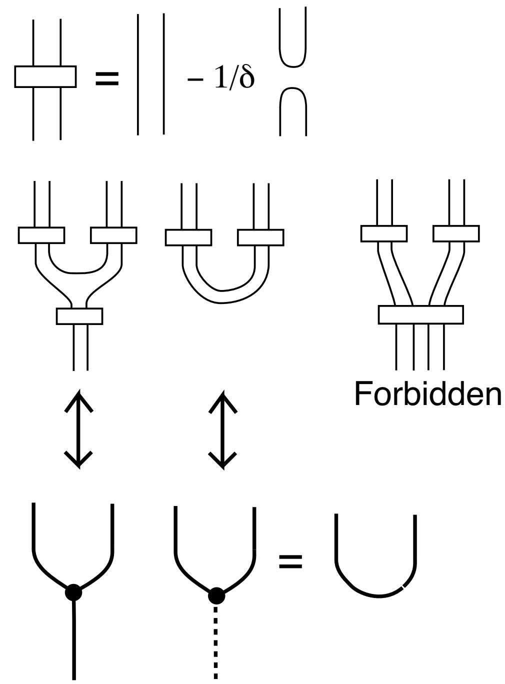
$$\Delta = \delta = (1 + \sqrt{5})/2.$$

$$F = \begin{pmatrix} 1/\Delta & 1/\sqrt{\Delta} \\ 1/\sqrt{\Delta} & -1/\Delta \end{pmatrix} = \begin{pmatrix} \tau & \sqrt{\tau} \\ \sqrt{\tau} & -\tau \end{pmatrix}$$

$$R = \begin{pmatrix} -A^4 & 0 \\ 0 & A^8 \end{pmatrix} = \begin{pmatrix} e^{4\pi i/5} & 0 \\ 0 & -e^{2\pi i/5} \end{pmatrix}.$$

Braid Representations Dense in Unitary Groups

$$A = e^{3\pi i/5}.$$



Temperley Lieb Representation of Fibonacci Model

Combination of Penrose Spin Networks and Knot Theory.

See "Temperley Lieb Recoupling Theory and Invariants of Three-Manifolds" by L. Kauffman and S. Lins, PUP, 1994.

Spin Networks and Anyonic Topological Computing

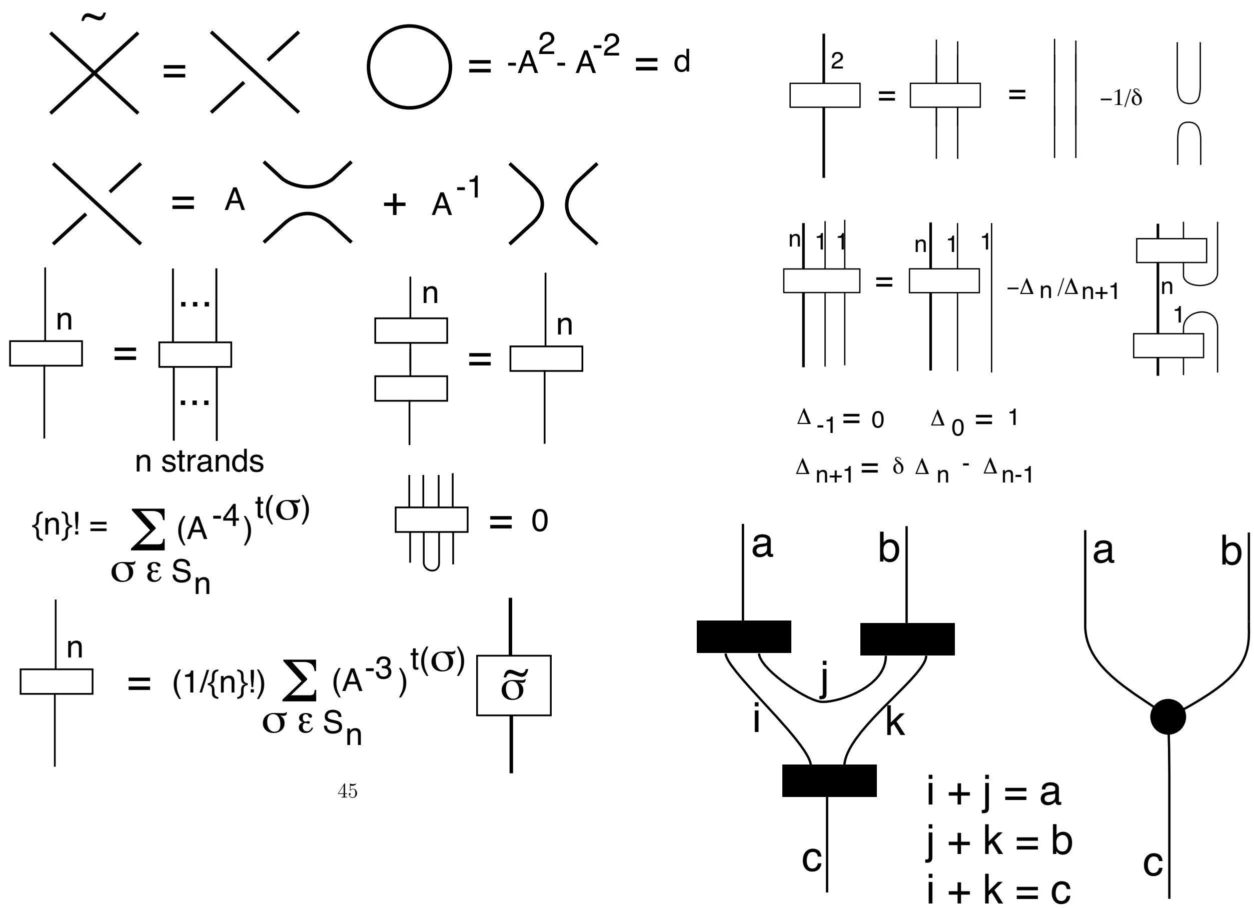
Louis H. Kauffman[a] and Samuel J. Lomonaco Jr.[b]

[a] Department of Mathematics, Statistics and Computer Science (m/c 249), 851 South Morgan Street, University of Illinois at Chicago, Chicago, Illinois 60607-7045, USA
[b] Department of Computer Science and Electrical Engineering, University of Maryland Baltimore County, 1000 Hilltop Circle, Baltimore, MD 21250, USA

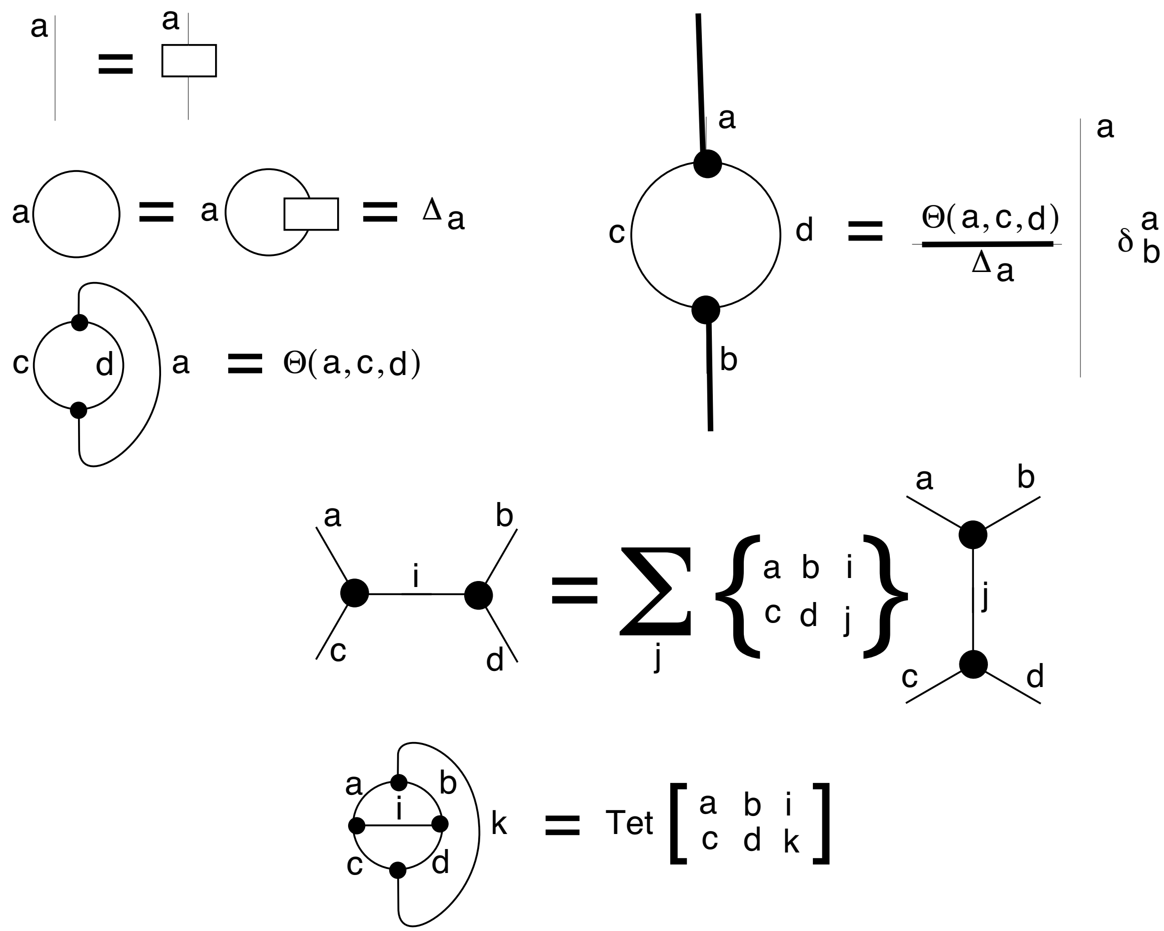
Recoupling Theory for Fibonacci Model is Joint work with Sam Lomonaco.

quant-ph/0603131 and quant-ph/0606114

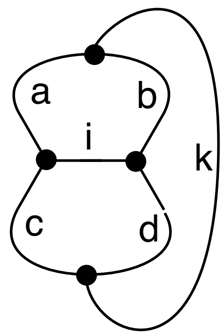
q-Deformed Spin Networks



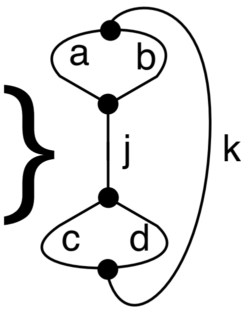
More Recoupling



The 6-j Coefficients



$$= \sum_j \begin{Bmatrix} a & b & i \\ c & d & j \end{Bmatrix}$$

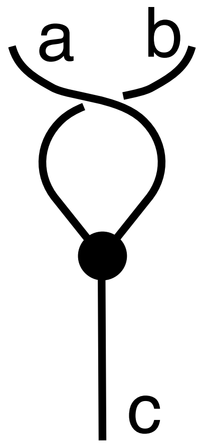


$$= \sum_j \begin{Bmatrix} a & b & i \\ c & d & j \end{Bmatrix} \frac{\Theta(a,b,j)}{\Delta_j} \frac{\Theta(c,d,j)}{\Delta_j} \Delta_j \, \delta_j^k$$

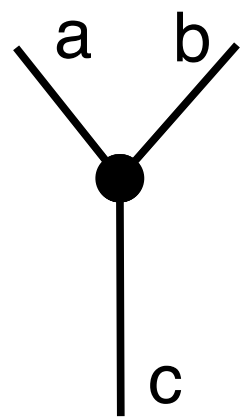
$$= \begin{Bmatrix} a & b & i \\ c & d & k \end{Bmatrix} \frac{\Theta(a,b,k)\, \Theta(c,d,k)}{\Delta_k}$$

$$\begin{Bmatrix} a & b & i \\ c & d & k \end{Bmatrix} = \frac{\mathrm{Tet}\begin{bmatrix} a & b & i \\ c & d & k \end{bmatrix} \Delta_k}{\Theta(a,b,k)\ \Theta(c,d,k)}$$

Local Braiding



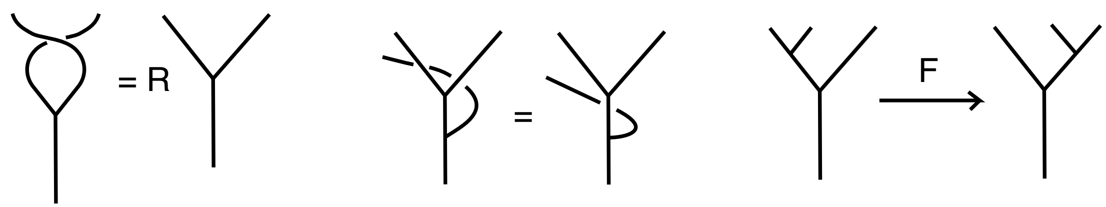
$$= \lambda^{ab}_{c}$$

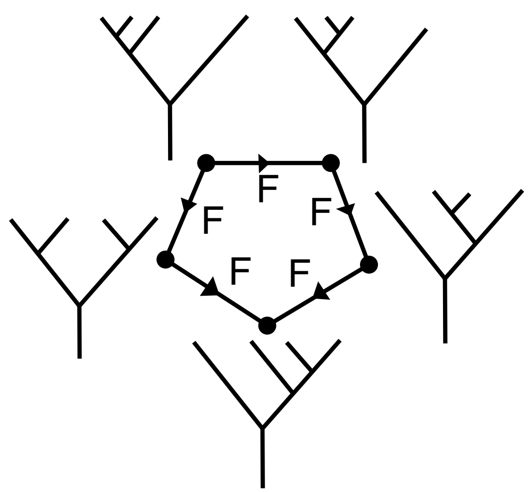


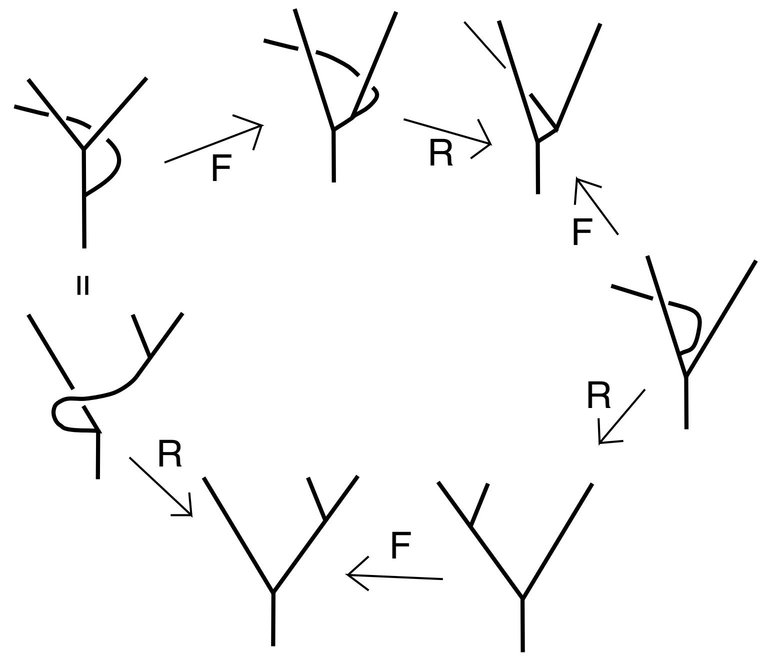
$$\lambda^{ab}_{c} = (-1)^{(a+b-c)/2} A^{(a'+b'-c')/2}$$

$$x' = x(x+2)$$

Braiding, Naturality, Recoupling, Pentagon and Hexagon -- Automatic Consequences of the Construction







Quantum Hall Effect

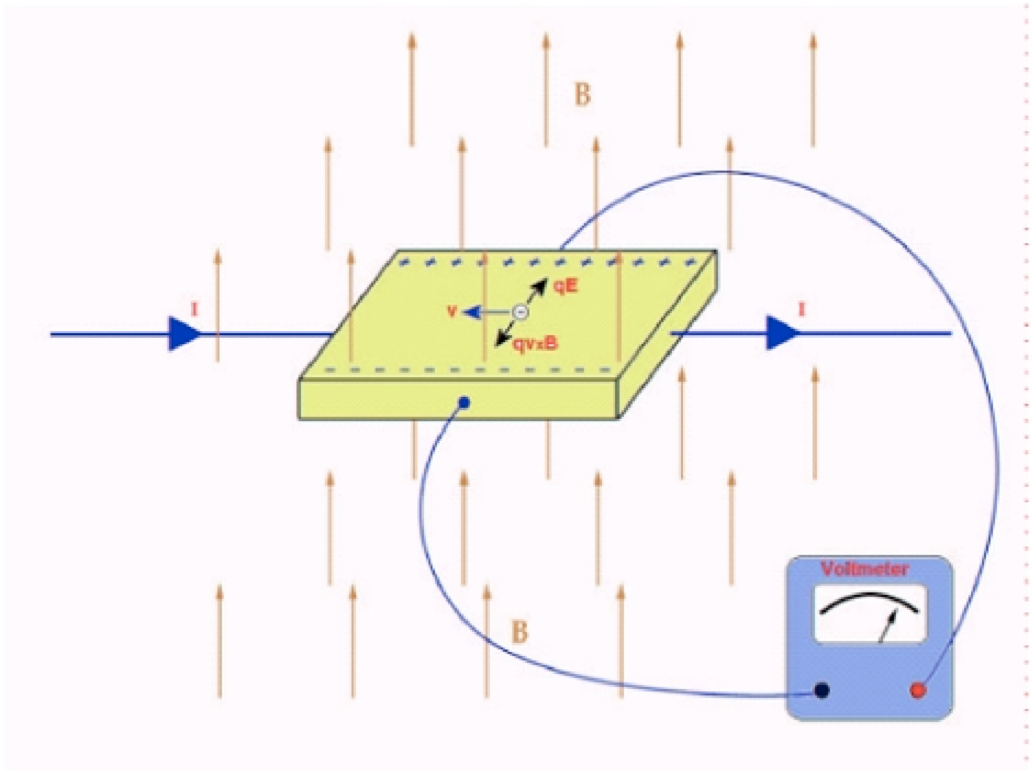
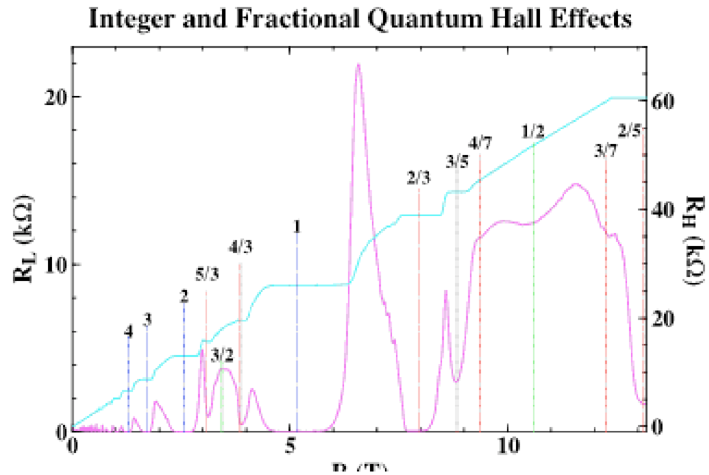


Figure 1: A schematics of the experimental setup of the Hall effect. A current driven through the conductor, drawn as a prism, leads to the emergence of voltage in the perpendicular direction. This is the Hall voltage, which Maxwell erroneously predicted to be zero.

Fractional Quantum Hall Effect (Cambridge Univ Website)

The fractional quantum Hall effect (FQHE) is a fascinating manifestation of simple collective behaviour in a two-dimensional system of strongly interacting electrons. At particular magnetic fields, the electron gas condenses into a remakable state with liquid-like properties. This state is very delicate, requiring high quality material with a low carrier concentration, and extremely low temperatures. As in the integer Quantum Hall Effect, a series of plateaux forms in the Hall resistance. Each particular values of magnetic field corresponds to a filling factor (the ratio of electrons to magnetic flux quanta) nu=*p*/*q*, where *p* and *q* are integers with no common factors). *q* always turns out to be an odd number. The principal series of such fractions are 1/3, 2/5, 3/7 etc, and 2/3, 3/5, 4/7, etc.



There are two main theories of the FQHE:

- **Fractionally-charged quasiparticles**. This theory, proposed by Laughlin, hides the interactions by constructing a set of quasiparticles with charge $e^*=e/q$, where the fraction is *p*/*q* as above.
- **Composite Fermions**. This theory was proposed by Jain, and Halperin, Lee and Read. In order to hide the interactions, it attaches two (or, in general, an even number) flux quanta *h*/*e* to each electron, forming integer-charged quasiparticles called Composite Fermions. The fractional states are mapped to the Integer QHE. This makes electrons at a filling factor 1/3, for example, behave in the same way as at filing factor 1. A remarkable result is that filling factor 1/2 corresponds to zero magnetic field. Experiments support this.

Nuclear Physics B360 (1991) 362–396
North-Holland

NONABELIONS IN THE FRACTIONAL QUANTUM HALL EFFECT

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Received 31 May 1990
(Revised 5 December 1990)

Appiications of conformal field theory to the theory of fractional quantum Hall systems are discussed. In particular, Laughlin's wave function and its cousins are interpreted as conformal blocks in certain rational conformal field theories. Using this point of view a hamiltonian is constructed for electrons for which the ground state is known exactly and whose quasihole excitations have nonabelian statistics; we term these objects "nonabelions". It is argued that universality classes of fractional quantum Hall systems can be characterized by the quantum numbers and statistics of their excitations. The relation between the order parameter in the fractional quantum Hall effect and the chiral algebra in rational conformal field theory is stressed, and new order parameters for several states are given.

1. Introduction

The past few years have seen a great deal of interest in two-dimensional many particle and $(2+1)$-dimensional field-theoretic systems from several motivations. These include the fractional quantum Hall effect, high-temperature superconductivity and the anyon gas, conformal field theory in $1+1$ dimensions and its relation to $2+1$ Chern–Simons–Witten (CSW) theories, knot invariants, exactly soluble statistical mechanical models in $1+1$ dimensions, and general investigations of

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(12) **Patent Application Publication**
Freedman et al.
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(43) **Pub. Date: May 4, 2006**

(54) **SYSTEMS AND METHODS FOR QUANTUM BRAIDING**

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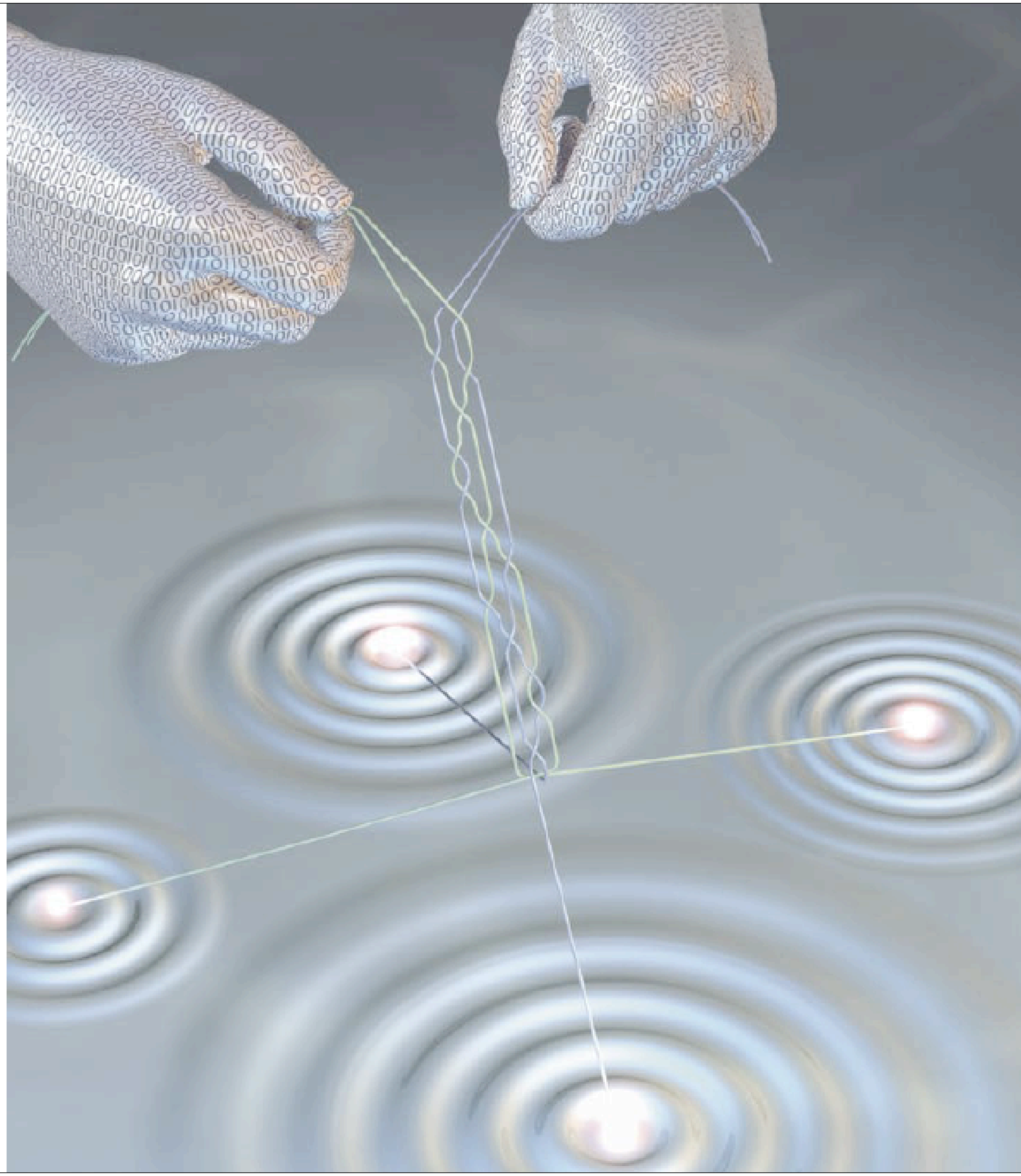
H01L 29/06 (2006.01)

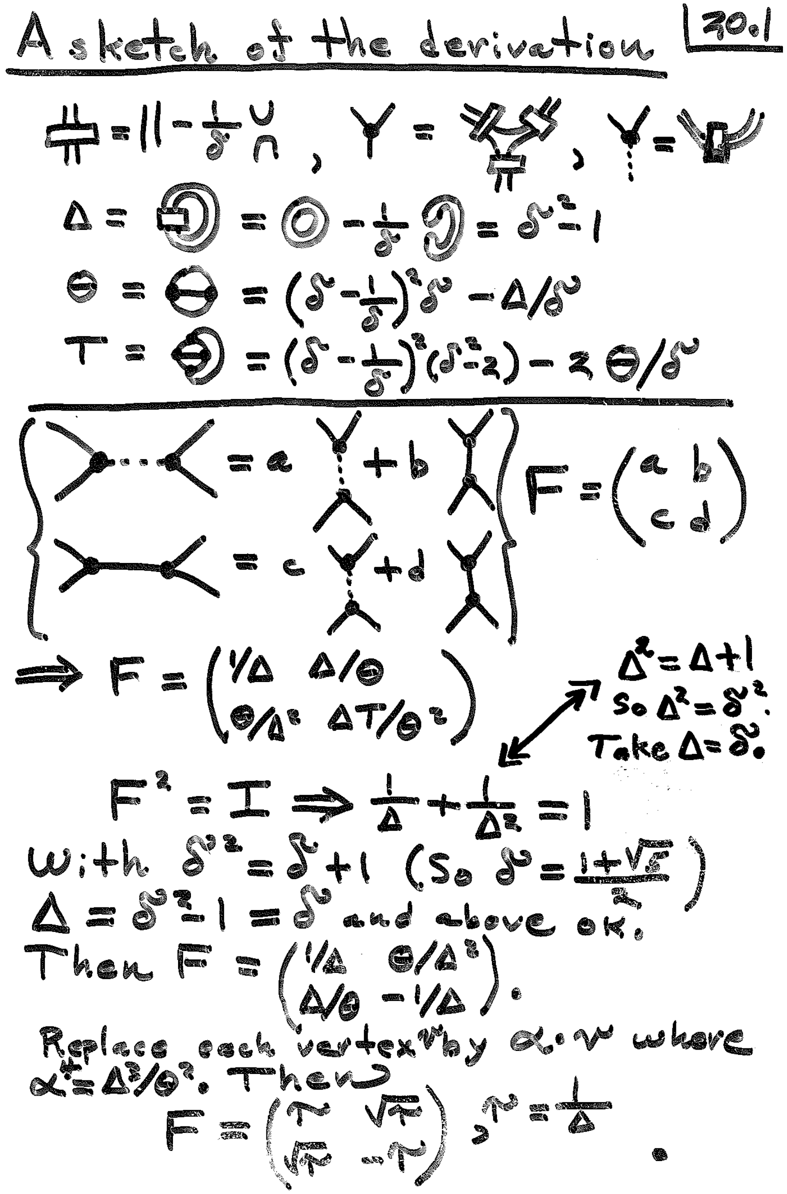
(52) **U.S. Cl.** **257/9**; 257/14

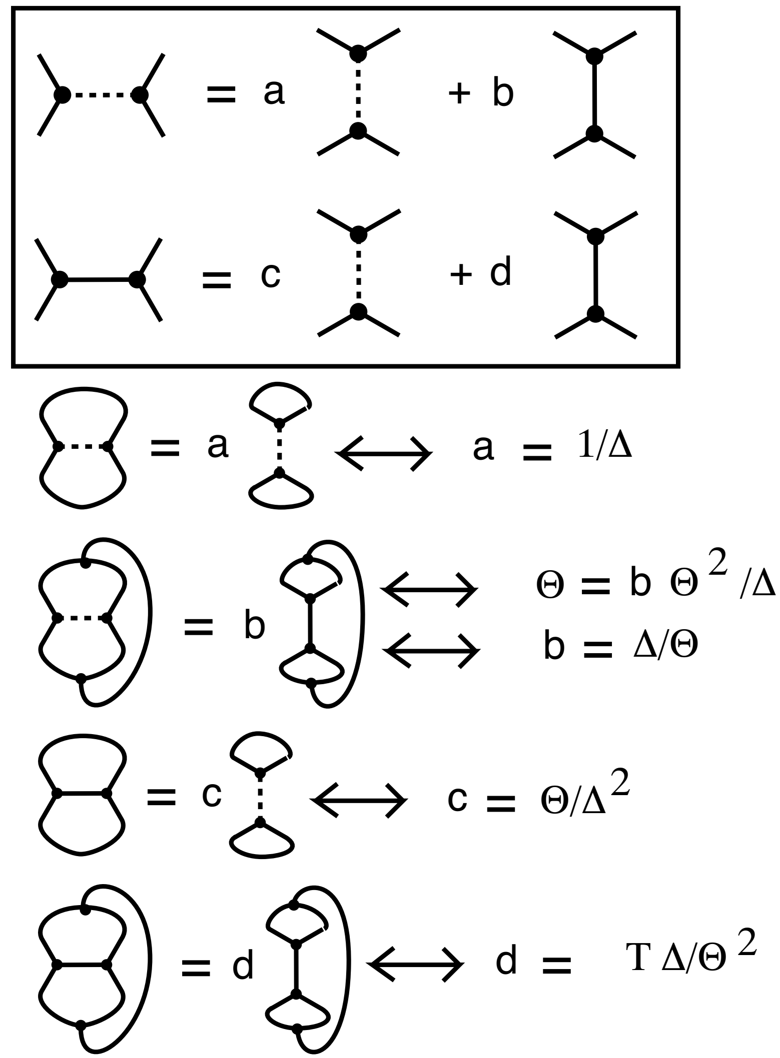
(57) **ABSTRACT**

Apparatus and methods for performing quantum computations are disclosed. Such apparatus and methods may include identifying a first quantum state of a lattice having a system of quasi-particles disposed thereon, moving the quasi-particles within the lattice according to at least one predefined rule, identifying a second quantum state of the lattice after the quasi-particles have been moved, and determining a computational result based on the second quantum state of the lattice. A topological quantum computer encodes information in the configurations of different braids. The computer physically weaves braids in the 2D+1 space-time of the lattice, and uses this braiding to carry out calculations. A pair of quasi-particles, such as non-abelian anyons, can be moved around each other in a braid-like path. The quasi-particles can be moved as a result of a magnetic or optical field being applied to them, for example. When the pair of quasi-particles are brought together, they may annihilate each other or create a new anyon. A result is that an anyon may be present or not, which can be thought of as a "one" or "zero," respectively. Such ones and zeros can be interpreted to provide information.



Fibonacci Model -- on the back of an envelope





Remark on the U(2) Representation

$$R = \begin{pmatrix} \mu & 0 \\ 0 & \lambda \end{pmatrix} \qquad F = \begin{pmatrix} a & b \\ b & -a \end{pmatrix}$$

$$R = \begin{pmatrix} \mu & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} + \begin{pmatrix} \mu - \lambda & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} + \lambda^{-1} \begin{pmatrix} \delta & 0 \\ 0 & 0 \end{pmatrix}$$

$$S = FRF = \lambda I + \lambda^{-1} V \text{ with } V = FUF.$$

$$V = FUF = \begin{pmatrix} a & b \\ b & -a \end{pmatrix} \begin{pmatrix} \delta & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ b & -a \end{pmatrix} = \delta \begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix}.$$

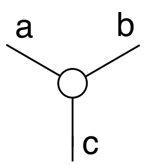
With these choices, we have

$$F = \begin{pmatrix} a & b \\ b & -a \end{pmatrix} = \begin{pmatrix} 1/\delta & \sqrt{1-\delta^{-2}} \\ \sqrt{1-\delta^{-2}} & -1/\delta \end{pmatrix}$$

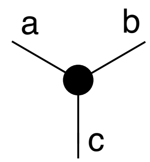
real and unitary, and for the Temperley-Lieb algebra,

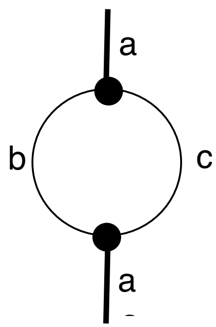
$$U = \begin{pmatrix} \delta & 0 \\ 0 & 0 \end{pmatrix}, \quad V = \delta \begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix} = \begin{pmatrix} a & b \\ b & \delta b^2 \end{pmatrix}.$$

Redefining the Vertex -- the key to obtaining Unitary Recoupling Transformations.

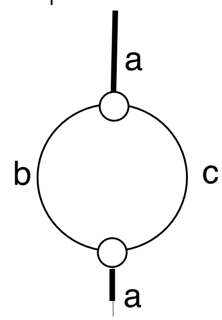


$$= \frac{\sqrt{\sqrt{\Delta_a \Delta_b \Delta_c}}}{\sqrt{\Theta(a,b,c)}}$$

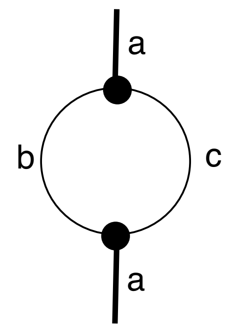


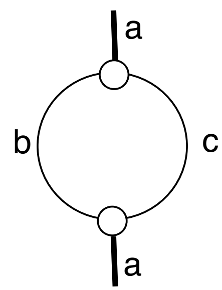


$$= \frac{\Theta(a,b,c)}{\Delta_a}$$

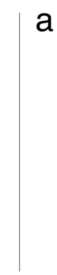


$$= \frac{\sqrt{\Delta_a \Delta_b \Delta_c}}{\Theta(a,b,c)}$$

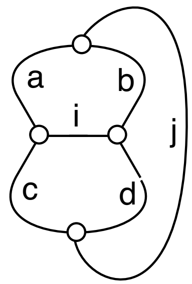




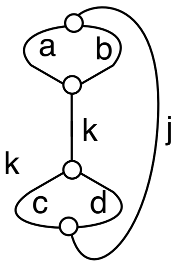
$$= \sqrt{\frac{\Delta_b \Delta_c}{\Delta_a}}$$



New Recoupling Formula



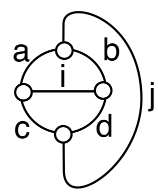
$$= \sum_k \boxed{\begin{matrix} a & b \\ c & d \end{matrix}}_{\,i\,k}$$



$$= \sum_k \boxed{\begin{matrix} a & b \\ c & d \end{matrix}}_{\,i\,k} \sqrt{\frac{\Delta_a \Delta_b}{\Delta_j}} \sqrt{\frac{\Delta_c \Delta_d}{\Delta_j}} \; \Delta_j \, \delta_j^k$$

$$= \boxed{\begin{matrix} a & b \\ c & d \end{matrix}}_{\,i\,j} \sqrt{\frac{\Delta_a \Delta_b}{\Delta_j}} \sqrt{\frac{\Delta_c \Delta_d}{\Delta_j}} \; \Delta_j$$

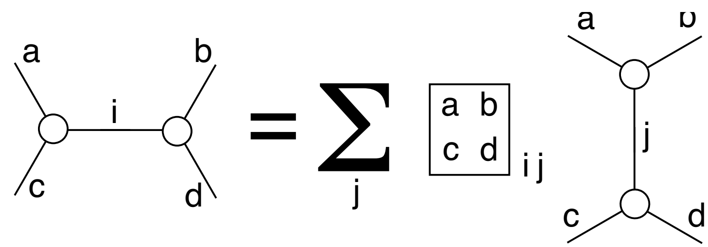
$$\boxed{\begin{matrix} a & b \\ c & d \end{matrix}}_{\,i\,j} =$$

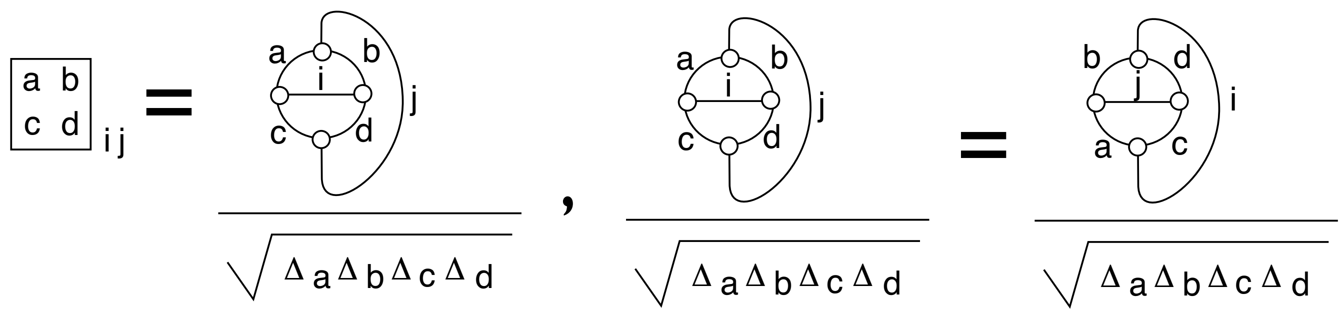


$$\overline{\sqrt{\frac{\Delta_a \Delta_b}{\Delta_j}} \sqrt{\frac{\Delta_c \Delta_d}{\Delta_j}} \; \Delta_j}$$

$$= \frac{\mathrm{ModTet}\begin{bmatrix} a & b & i \\ c & d & j \end{bmatrix}}{\sqrt{\Delta_a \Delta_b \Delta_c \Delta_d}}$$

The Recoupling Matrix is Real Unitary at Roots of Unity.





$$M[a,b,c,d]_{ij} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{ij}$$

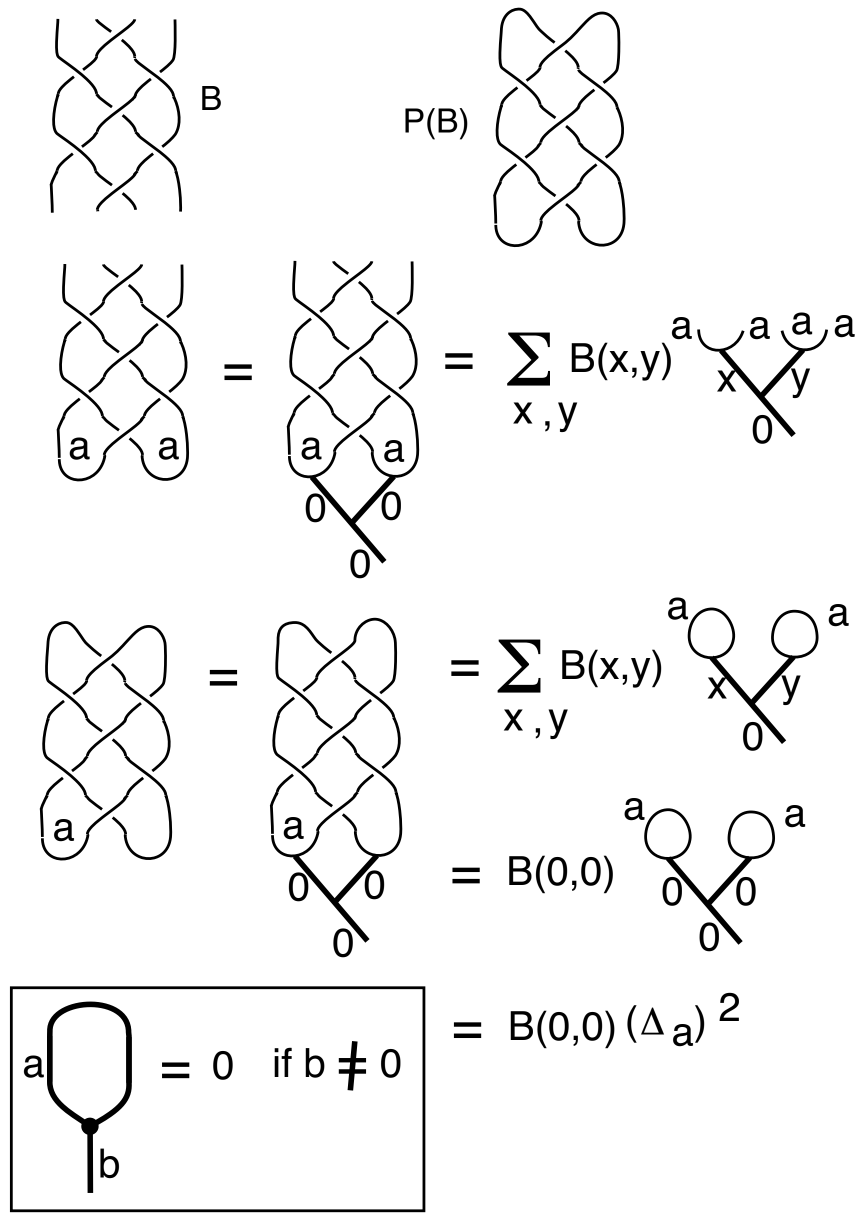
$$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{T} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$$

Theorem. Unitary Representations of the Braid Group come from Temperley Lieb Recoupling Theory at roots of unity.

$$A = e^{i\pi/2r}$$

Sufficient to Produce Enough Unitary Transformations for Quantum Computing.

Quantum Computing Colored Jones Polynomials



Computing the colored Jones polynomials at roots of unity requires finding a single diagonal element of a unitary matrix.

The best quantum algorithm we know for this is the Hadamard test. (See next slides.)

Aharanov, Jones and Landau also use the Hadamard test in their algorithm for the Jones polynomial. Computation time for our algorithm and theirs are the same -- polynomial time for numerical approximation of the values of the invariant.

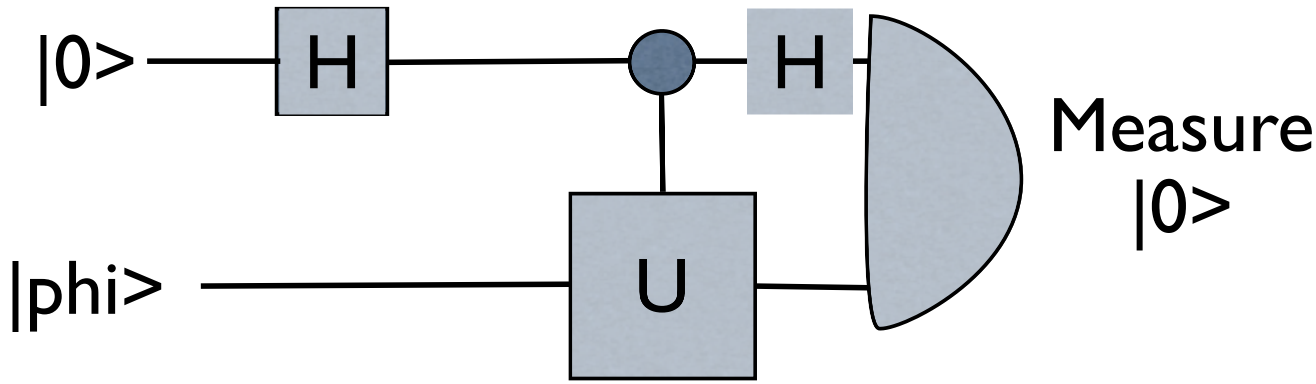
Witten-Reshetikhin-Turaev Invariants

WRT invariants of three manifolds
are obtained by special sums of
colored Jones polynomials.
Thus we also implicitly give algorithms for
computing WRT invariant.

What does this have to do with the
quantum field theory associated with
Witten's approach?

Is there a direct quantum algorithm for
the Witten functional integral?

Hadamard Test



$$H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} / \mathrm{Sqrt}(2)$$

|0> occurs with probability
1/2 + Re[<phi|U|phi>]/2.

Imaginary part by same circuit with a phase shift of Pi/2.

Summary

The simple Fibonacci model is universal for quantum computing. All quantum mechanical processes can be simulated by this model.

The Fibonacci model is constructed from the bracket model for the Jones polynomial.

The Fibonacci model is constructed by a logic that goes beneath the Boolean stucture implicit in a space of one qubit -- allowing the action of the three-strand braids on the qubit space.

The Fibonacci model and its relatives
show that in principle quantum computing
can be accomplished with topological means.

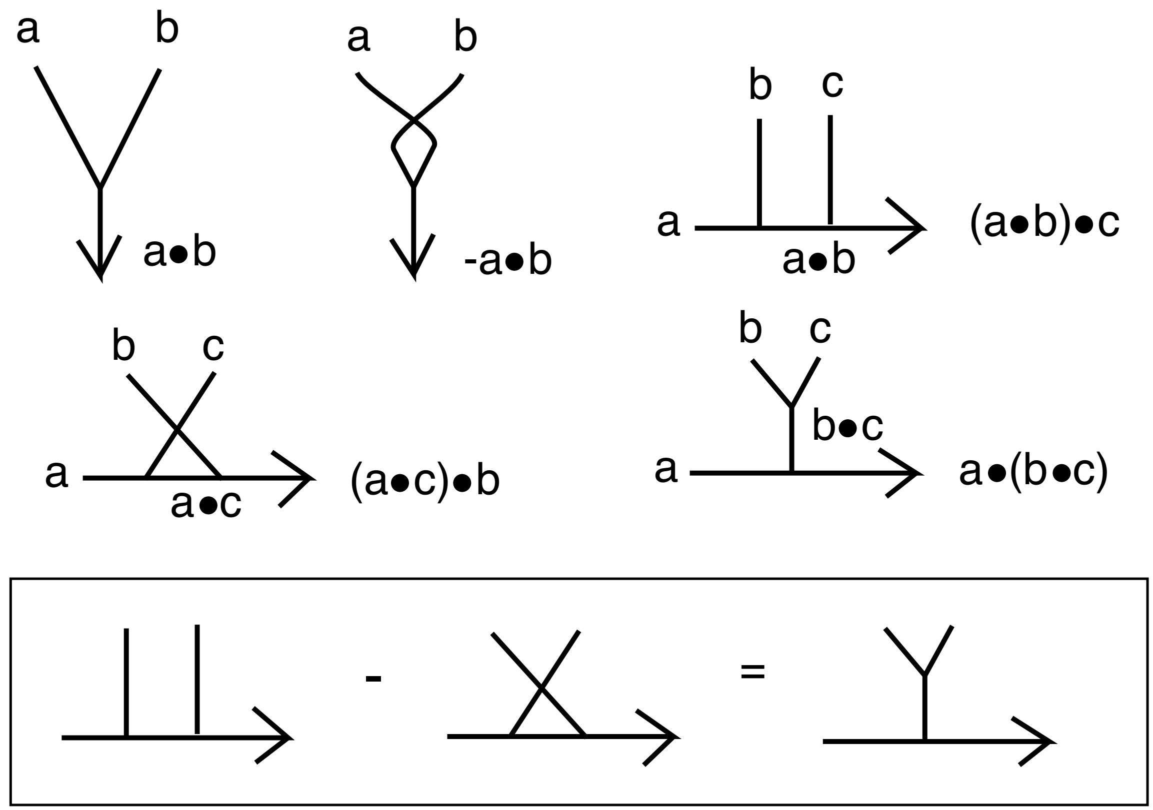
The theory of the quantum Hall effect suggests
that non-abelian anyons can realize this dream.

Will the dream come to pass?

And for the mathematician -- what is the depth
of the role of the Artin braid group in the
structure of the unitary groups?

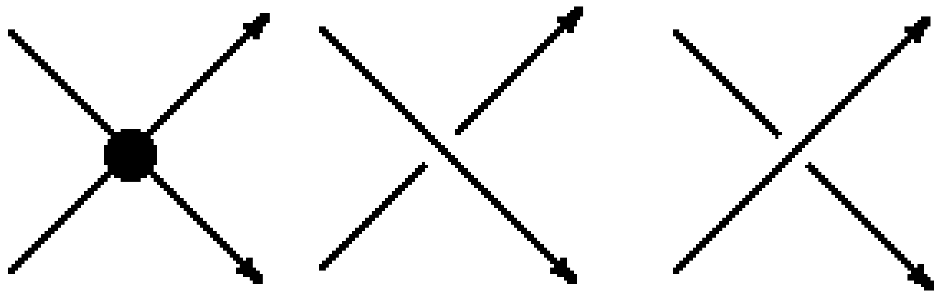
Returning to Frege

The Jacobi Identity



(a•b)•c - (a•c)•b = a•(b•c)
Hence
(a•b)•c + b•(a•c) = a•(b•c).

Knots, Links and Lie Algebras
Vassiliev Invariants

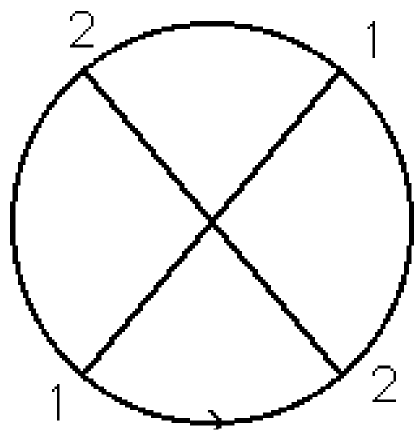


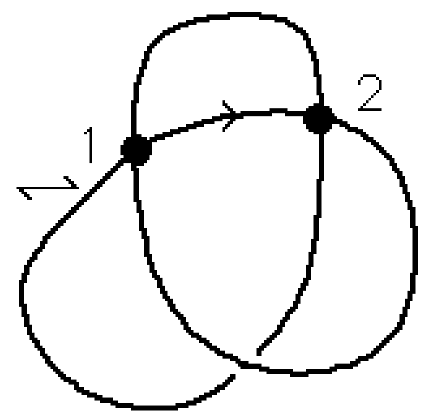
(K|*) (K|+) (K|-)

v(K|*) = v(K|+) - v(K|-)

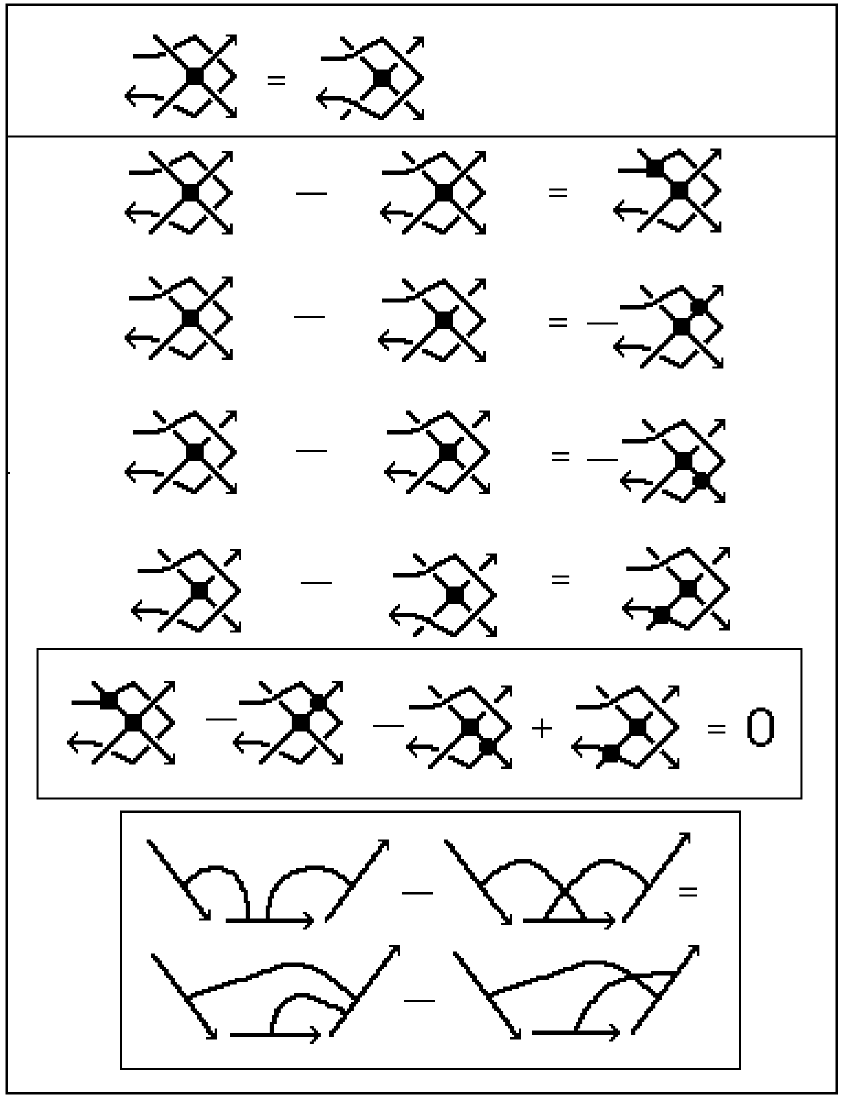
Skein Identity

Chord Diagram

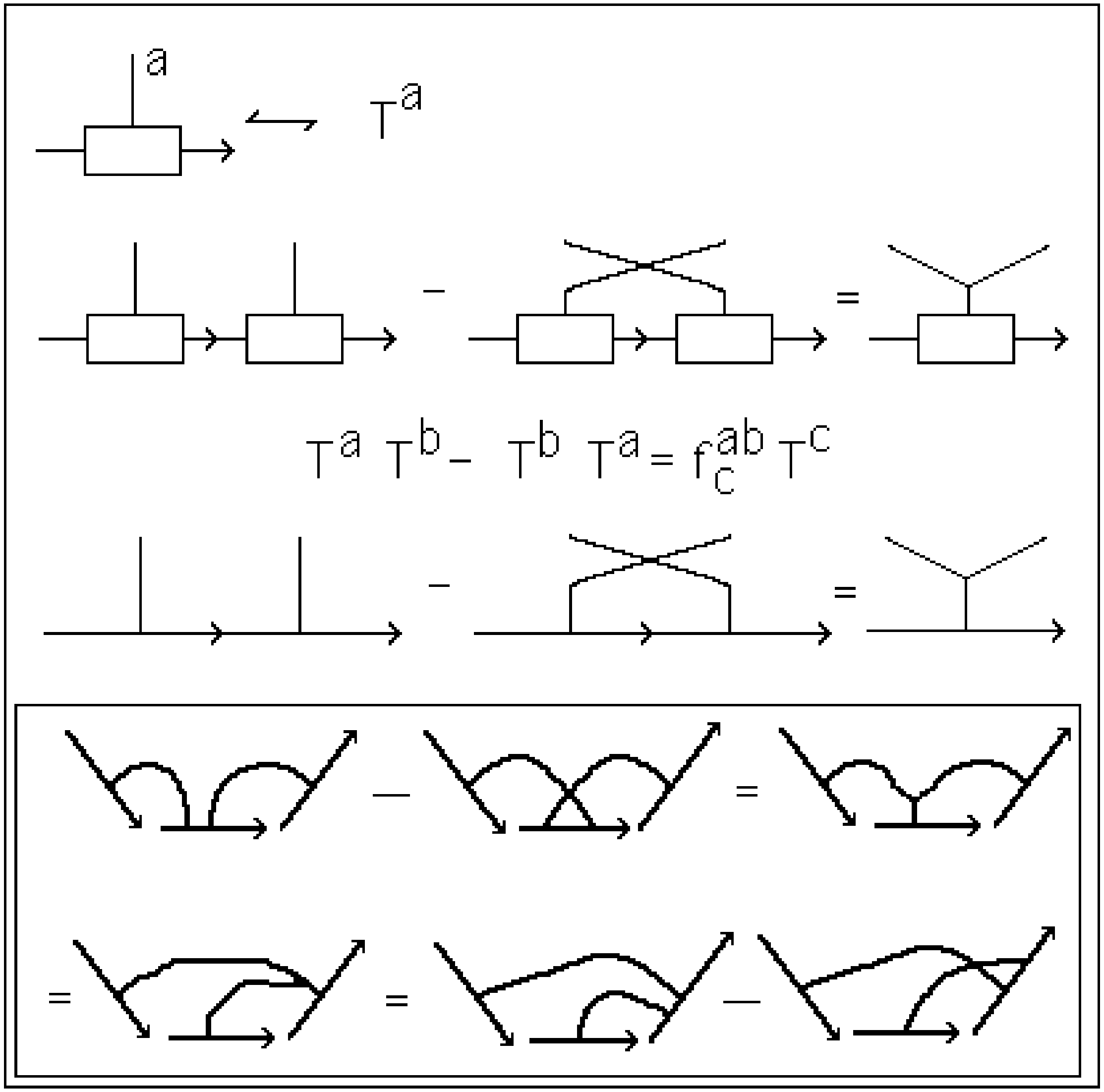




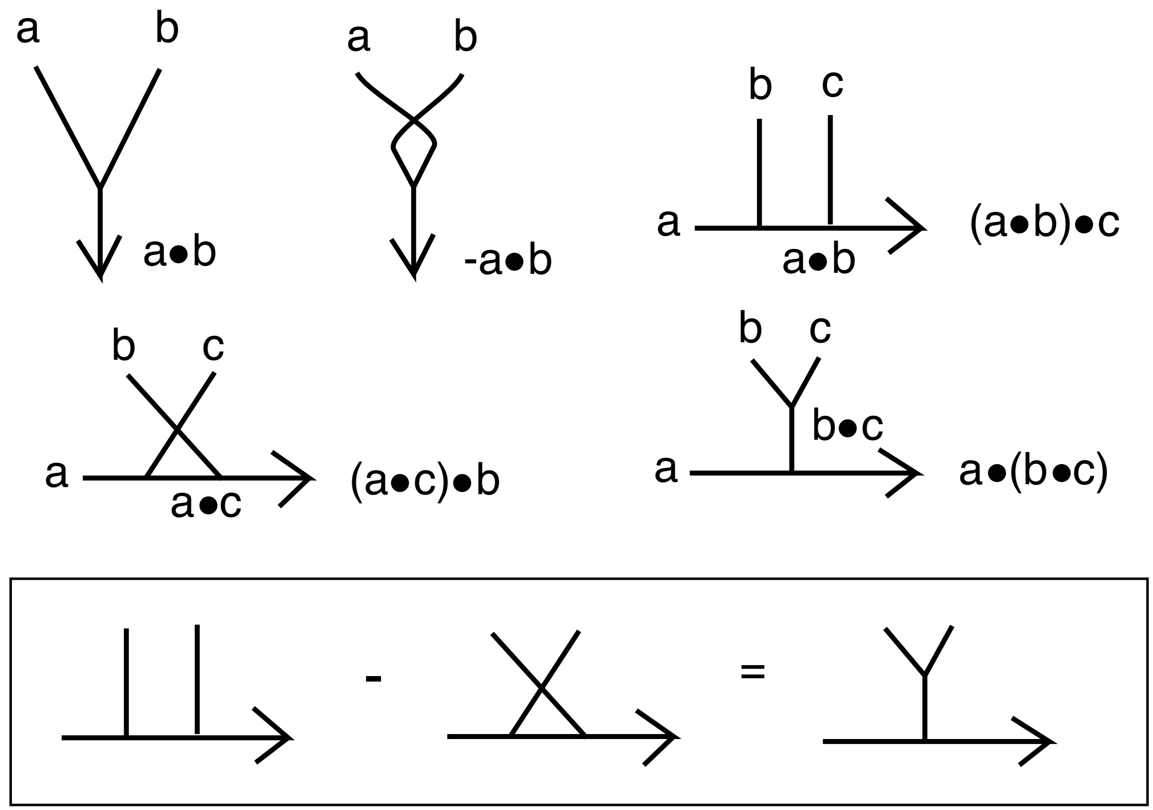
Four-Term Relation From Topology



Four Term Relation from Lie Algebra



The Jacobi Identity



(a•b)•c - (a•c)•b = a•(b•c)

Hence

(a•b)•c + b•(a•c) = a•(b•c).

Knot theory, logic and physics all fit together in the categorical diagrammatic setting.

The story goes on.