A BALANCE GAME

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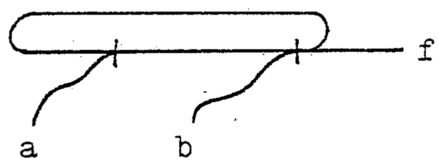
The purpose of this note is to present a way to interpret the re-entry (self-referential) expressions of G. Spencer Brown in Laws of Form. We shall assume that the reader is familiar with Brown's book. A more detailed version of our viewpoint will appear in the International Journal of General Systems in a paper titled "Network Synthesis and Varela's Calculus."

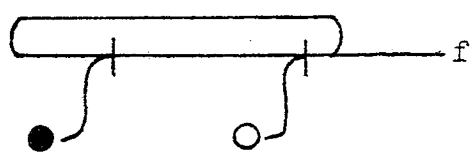
Let's begin with the expression f = $\overline{\overline{fa}|\,b}|$. In Brown's graphical notation it has the form



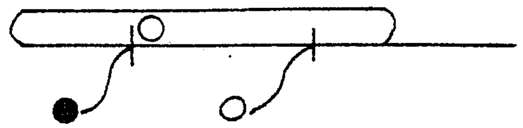
Let ● denote the marked state and ○ denote the unmarked state. In a network like this we shall say that it is balanced at a given mark (—┼—) if whenever the marked state is indicated on any lead coming into a mark then the unmarked state is indicated on the right-hand side of this same mark and if whenever all inputs to the mark are unmarked then the marked state is indicated on the right-hand side. Thus

●——┼○—— and ○——┼●—— are balanced. But ○——┼○—— and ●——┼●—— are unbalanced.

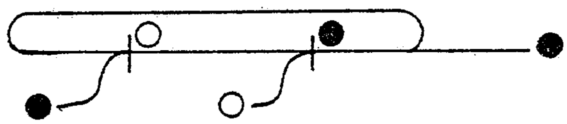
Thus suppose that a = ⌉ and b = ⌉⌉ are given and held at these values.



Then in order to obtain balance we must indicate the unmarked state to the right of the left-most mark:

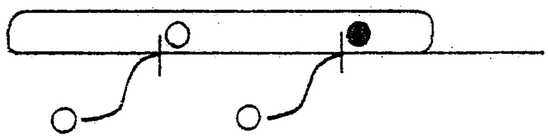


But now the inputs to the mark on the right are both unmarked and therefore balance requires us to put a marked state at the right-hand mark:



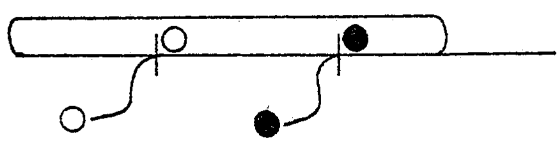
The network is now balanced since the re-entry does not create any new imbalance.

Furthermore, if we let a change to the unmarked state, no imbalance is introduced. The expression <u>remembers</u> a = ⌉ , b = ⌉⌉ :

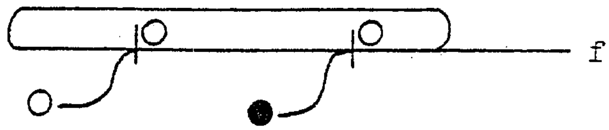


If <u>all</u> the marks in a network are balanced, then we say that the <u>net is balanced</u>.

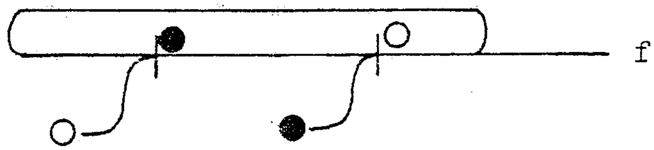
Note that if b is now changed to the marked state, then the situation becomes



This is unbalanced at the right-hand mark. We correct the imbalance:

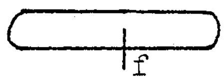


But now it is unbalanced on the left and so, correcting again:

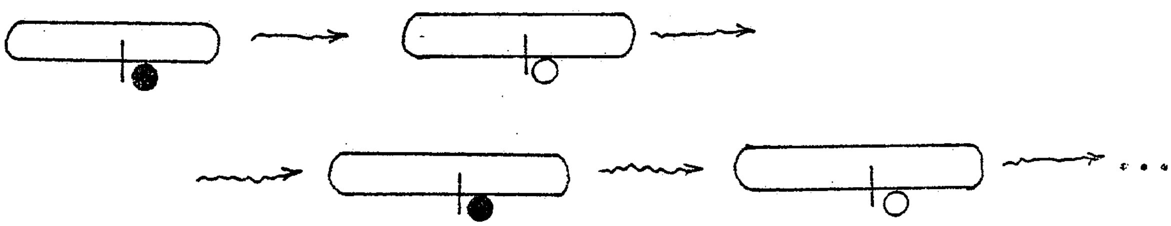


Now it is balanced and if b becomes unmarked, it will remember this new state.

Here is a second example. Let f = $\overline{f}\rceil$:



This is never balanced. Each attempt to restore balance just flips it to its other unbalanced state:



Thus it oscillates between marked and unmarked states.

The balance game is a way to watch the time-behavior of Brownian expressions. Here are the rules:

1) Draw a large diagram of the network.

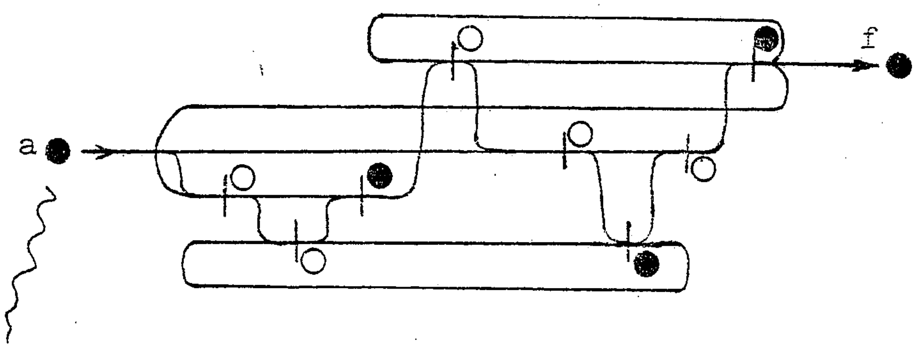
2) Place black and white counters on the inputs and just to the right of each mark. If your choice of counters gives an unbalanced state follow rule 3 to search for balance.

3) Without changing any inputs, find the unbalanced marks. Choose one. Balance it by changing its counter.

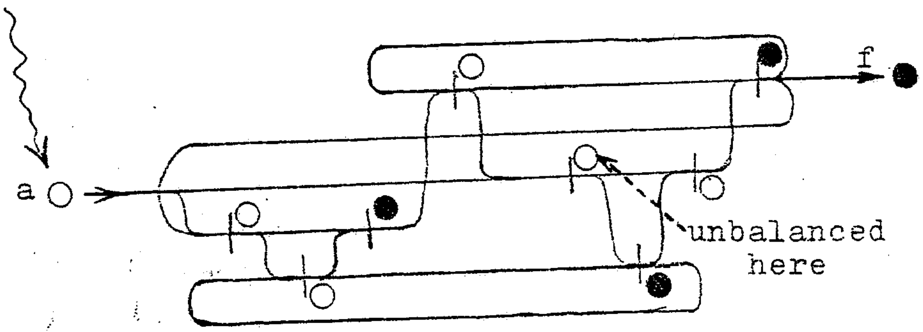
Now there may be a new set of unbalanced marks. If so, choose again and keep doing this until the net is balanced (or until it is clear that no balance can be obtained).

4) Suppose the net is balanced. Then change an input. Now return to step 3) and play out the new transition to balance. Keep a record of the balanced states and transitional behavior.

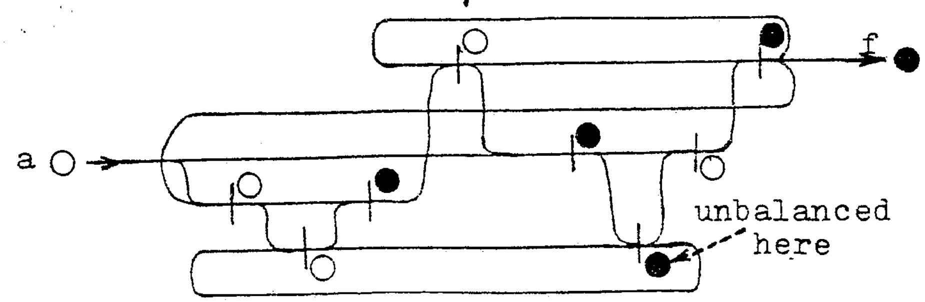
This procedure provides a good way to see how the modulators of Brown's Chapter 11 work. As an example we illustrate one transition in the first Brownian modulator:

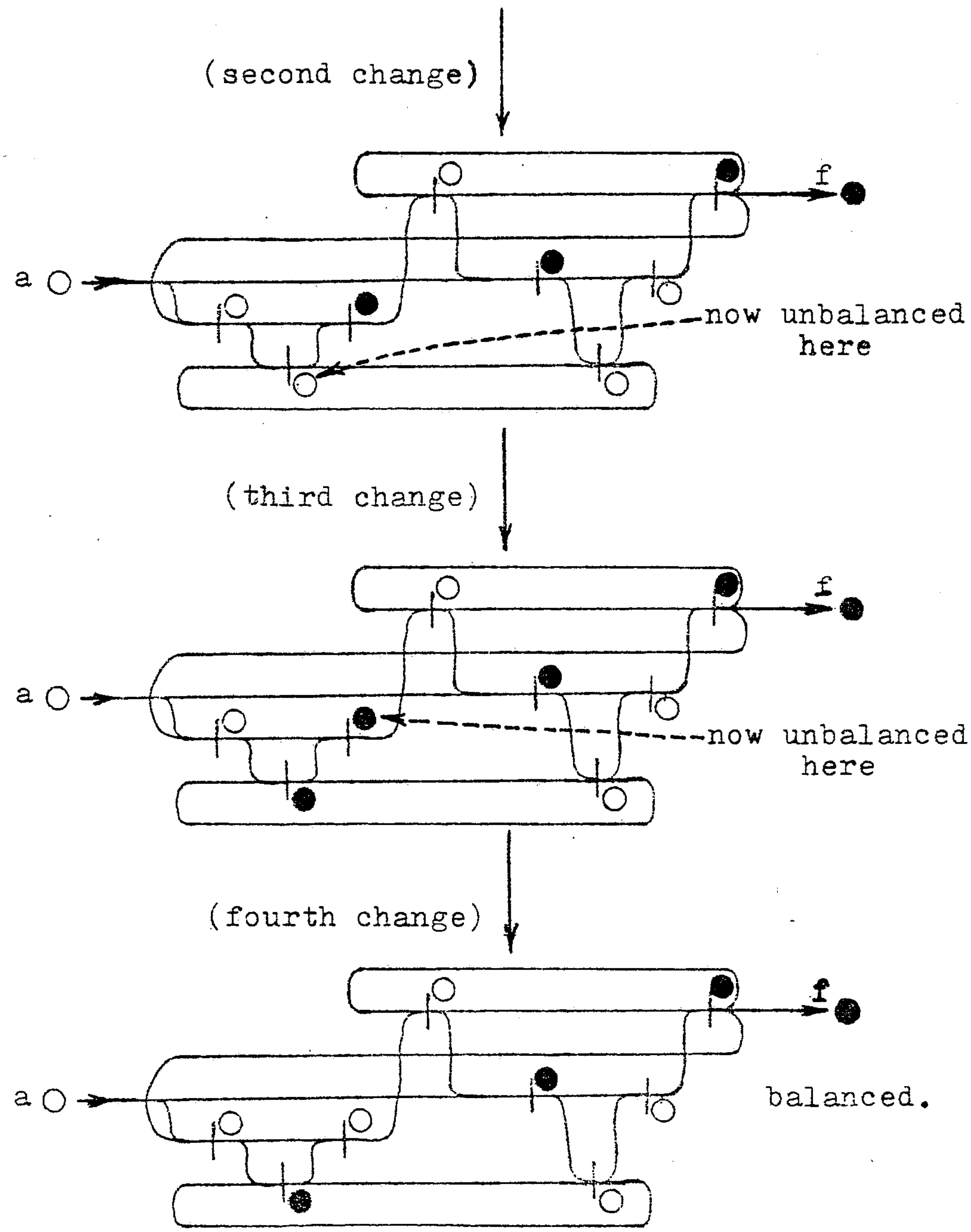


This is balanced. Let a become unmarked.



(first change)





Thus the network moves to a new balanced state when a is changed. These transitions are very easy to work through using counters (say Go stones) on a circuit diagram. They illustrate the delicate communicational relationships that give the network its behavior.

Note how time enters in this process. If the marks have different reaction times this does not affect the behavior as long as the perturbation of the input a is not too fast. Thus in a certain sense, the behavior is timeless, although any particular realization will have the imperfections of confusion in reaction to overly-rapid change (perturbation).

Balance and change: In balance there is a fitting-together—hand-in-glove—the sound of one hand clapping. Push. Perturb. Then comes the rush of self-correction, from the swinging of a pendulum or the internal echoes of a Chinese gong, to the clouds of passing thoughts. This reverberation may return to a quiescent state. Or there may be a continued sounding, a presence of form.

Logical Form

Volume I Number 1

May 1977