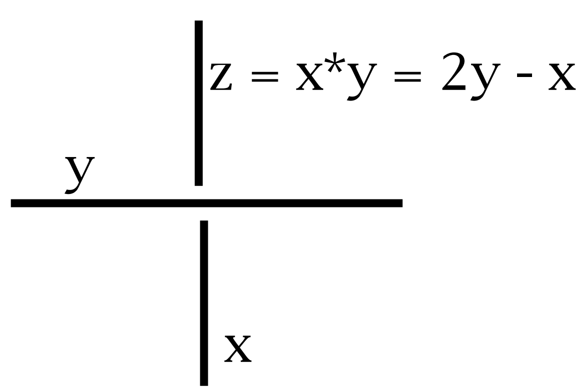
Homework Number 2 - Math 569

1. Let $x*y = 2y - x$ for x and y in Z or in Z/NZ for some modulus N. Show that this operation satisfies the involutory quandle axioms:
(a) $x*x = x$
(b) $(x*y)*y = x$
(c) $(x*y)*z = (x*z)*(y*z)$.
For Z/NZ the binary operation * gives an action of each residue class k on the set $\{0,1,\ldots, N-1\}$ via x -----> x*k. Let p(k) denote this permutation. Show that the set of permutations so obtained generates the dihedral group D_{2N} of symmetries of a regular N-gon.

2. Let G be any group with multiplicative binary operation. Define $g*h = hg^{-1}h$ for g and h in G. Show that * satisfies the axioms for an involutory quandle.

3. Recall that we have defined the quandle by using two binary operations. For this word processor I will use x*y and x#y for the two operations. Then the quandle axioms are:
(a) $x*x = x$, $x\#x=x$
(b) $(x*y)\#y = x = (x\#y)*y$
(c) $(x*y)*z = (x*z)*(y*z)$, $(x\#y)\#z = (x\#z)\#(y\#z)$.
Show that if M is a module over $Z[t,t^{-1}]$, and we define
$a*b = ta + (1-t)b$
$a\#b = t^{-1}a +(1-t^{-1})b$,
then this gives M the structure of a quandle.

4. Suppose that you can label the arcs of a knot diagram with some elements of Z/NZ so that the relation $z = 2y -x$ is satisfied at every crossing.



Show that you can then represent the fundamental group of the knot to the dihedral group D2N (see exercise 1) by sending the element of the fundamental group corresponding to each arc of the diagram in the Wirtinger presentation to the permuation p(x) corresponding to the color x on that arc. Recall the definition of p(x) from exercise 1. Apply your result explicitly to the trefoil knot and to the figure eight knot.