165testthreesample2009.mw
Maple 10 Worksheet for Problems in Math 165 - Calculus for Business.
N.B. These problems are a lot easier to do "by hand"
First load plots and student including student:-Calculus1

N.B. A Maple command such as eval(f(x),x=2) is the instruction
``Evaluate f(2)" or
``evaluate the function f(x) at x = 2."
a:= b assigns the value 'b' to the name 'a'
Use a semicolon ; to execute a 'Maple Command' and display the output.
Using a colon : executes a command without displaying the output.
I (JL) usually write functions as a procedure such as the "x square funtion:
square_function:= proc(x);x^2 ; end proc;
Other ways to write functions:
square_function:= x -> x^2;
`%` is the last computed expression (Simlar to ANS on your calculator).

Maple procedure to emulate TI fnInt

$$fnInt := \textbf{proc}(f, X, A, B)\ int(f(X), X = A..B)\ \textbf{end proc}$$

$$y1 := x \rightarrow x^2$$

$$TI\ says\ fnInt(y1,x,0,1.) := 0.3333333333 \qquad (1)$$

> 1. Suppose $1,500 is invested at an annual interest rate of 8 percent compounded quarterly. Compute the balance after 12 years.
A) $3,780.61
B) $3,820.61
C) $3,880.61
D) $3,890.61

This is NOT the continuous case
Cont_Ans_1:1500*exp(.08*12);
Quart_Ans_1: 1500*(1 + .08/4)^(4*12);

$$Cont\_Ans\_1 := 3917.544710$$

$$Quart\_Ans\_1 := 3880.605578 \qquad (2)$$

Now the contiuous case
2. How much money should be invested today at an annual interest rate of 9% compounded continuously Jso that 30 years from now it will be worth $27,000?
A) $ 24,676.14
B) $ 2,035.02
C) $ 401,752.76
D) $ 1,814.55

PV:= 27000*exp(-.09*30);

$$PV := 1814.548844 \qquad (3)$$

3. A radioactive substance decays exponentially. If 800 grams were present initially and 600 grams are present 100 years later, how many grams will be present after 400 years?
A) 251.93 grams
B) 251.97 grams
C) 252.01 grams
D) 253.13 grams

Multiply by (600/800) every 100 years.

$$RS(t) := 800 \left(\frac{3}{4}\right)^{\frac{1}{100}t}$$

$$answer := 253.1250000 \quad (4)$$

4. Solve the given equation for x. 1 = 9 e^(-2x)

Note that ln 9 = 2 ln 3 so that ln3 = (ln 9)/2

$$eqn := -8 = -9 + 9\,e^{-2x}$$

$$answer := \ln(3) \quad (5)$$

5. A radioactive substance decays exponentially. If 700 grams were present initially and 200 grams are present 100 years later, how many grams will be present after 400 years?

A) 4.66 grams
B) 0 grams
C) 3.41 grams
D) 2.16

Multiply by 200/700 every 100 years.

$$RS(t) := 700 \left(\frac{2}{7}\right)^{\frac{1}{100}t}$$

$$answer := 4.664723031 \quad (6)$$

6. The equation of the tangent line to y = e^(x^2)at x = 2

Differentiate 2x e^(x^2);slope at x =2 is 4 e^4; y - e^4 = 4 e^4 (x - 2)

$$\textit{Tangent line at x = 2 is formula} := 4\,x\,e^4 - 7\,e^4 \quad (7)$$

7.Find dy/dx for 20 - 5e^(-0.03x)

$$y1(x) := 20 - 5\,e^{-0.03\,x}$$

$$diff\_y1 := 0.15\,e^{-0.03\,x} \quad (8)$$

8. A manufacturer can produce radios at a cost of $10 apiece and estimates that if they are sold for x dollars apiece, consumers will buy approximately 200*exp(-0.2*p) radios per month.

The price at which the manufacturer should sell the radios to maximize the profit is

A) $10
B) $15
C) $18
D) $20

$$Demand(p) := 200\,e^{-0.2\,p}$$

$$Profit(p) := 200\,(p-10)\,e^{-0.2\,p}$$

$$deriv\_Profit(p) := 200\,e^{-0.2\,p} - 40.0\,(p-10)\,e^{-0.2\,p}$$

$$crit\_number := 15. \quad (9)$$

9. The consumer demand for a certain commodity is D(p) =5000*exp(-0.03*p)
units per month
when the market price is p dollars per unit. Determine the market price that will result in the greatest consumer expenditure.

A) $30.31
B) $31.31
C) $33.33
D) $34.33

$$Demand(p) := 5000\, e^{-0.03\, p}$$

$$Revenue(p) := 5000\, p\, e^{-0.03\, p}$$

$$deriv\_Revenue(p) := 5000\, e^{-0.03\, p} - 150.00\, p\, e^{-0.03\, p}$$

$$crit\_number := 33.33333333 \qquad (10)$$

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10. Find the derivative of ln((ln(x^2))^5)

Use ln rules to simplify; NO CORRECT ANSWER answer is 5/(x*(ln x))

$$y1(x) := \ln\left(\ln\left(x^2\right)^5\right)$$

$$answer := \frac{10}{\ln\left(x^2\right)\, x} \qquad (11)$$

11. The equation of the tangent line to

Kind of a mess;
Note 6^5 = 7776; Differentiate; (5x^4)e^(x^5);slope at x =6 is 5*6^4*e^(7776); ...

$$\textit{Tangent line at x = 2 is formula} := 6480\, x\, e^{7776} - 38879\, e^{7776} \qquad (12)$$

12. Find the critical numbers for 8 * x^3 * exp(8*x)

Notice that you are just finding the roots of the "coefficient" of e^(8x). x= 0 is a double critical number. (some difficulty not using procedures in Maple JL)

$$y1(x) := 8\, x^3\, e^{8\, x}$$

$$deriv\_y1(x) := 24\, x^2\, e^{8\, x} + 64\, x^3\, e^{8\, x}$$

$$ans := -\frac{3}{8}, 0, 0 \qquad (13)$$

13. Evaluate integral of 5*x^3 - 3*x +4

$$y1(x) := 5\, x^3 - 3\, x + 4$$

$$Ans := \int \left(5\, x^3 - 3\, x + 4\right) \mathrm{d}x$$

$$ans := \frac{5}{4}\, x^4 - \frac{3}{2}\, x^2 + 4\, x + C \qquad (14)$$

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14. Find the function whose tangent line has the slope 3*x^2 +1 for each value of x and whose graph passes through (0, 2).

$$y1(x) := 3\, x^2 + 1$$

$$Answer := 2 + \int_0^x \left(3\, t^2 + 1\right) \mathrm{d}t$$

$$answer := 2 + x^3 + x \qquad (15)$$

15. A study indicates that x months from now the population of a certain city will be increasing at the rate of (3 + 4*x)*x^(-1/2)
people per month. By how much will the
population increase over the next 9 months?
A) 70 people
B) 80 people
C) 90 people
D) 100 people

Looking for total change in 9 months Actually an improper integral!

$$rate(t) := \frac{3+4t}{\sqrt{t}}$$

$$Answer := \int_0^9 \frac{3+4t}{\sqrt{t}}\,dt$$

$$answer := 90 \qquad (16)$$

16. A manufacturer makes a certain product at a rate of t^2 -3*t +5 items per hour. How many items does the company make on average during the second hour?
A) 2.83
B) 11.83
C) 4.83
D) 10.83

Notice that SECOND HOUR is actually t from 1 to 2

$$rate(t) := t^2 - 3t + 5$$

$$Answer := \int_1^{2.} \left(t^2 - 3t + 5\right)\,dt$$

$$answer := 2.833333333 \qquad (17)$$

17. Evaluate integral of (Hard to Read). Answer A. corresponds to: 9*x^7 - 7*x +8

$$y1(x) := 9x^7 - 7x + 8$$

$$Ans := \int \left(9x^7 - 7x + 8\right)\,dx$$

$$ans := \frac{9}{8}x^8 - \frac{7}{2}x^2 + 8x + C \qquad (18)$$

18. Specify the substitution you would choose to evaluate the integrals.
of sqrt(4 - 2*t)
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Actually,all are valid, but only u = 4 - 2t is really useful.

$$y1(t) := \sqrt{4-2t}$$

$$Problem := \int \sqrt{4-2t}\,dt$$

$$result := \frac{2}{3}(-2+t)\sqrt{4-2t} + C$$

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Creating problem #6
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Applying substitution t = -1/2*u+2, u = 4-2*t with dt = -1/2*
du, du = -2*dt
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$$\int \sqrt{4-2\,t}\;\mathrm{d}t = \int \left(-\frac{1}{2}\,\sqrt{u}\right)\mathrm{d}u$$

```
Reverting substitution using u = 4-2*t
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$$answer := \int \sqrt{4-2\,t}\;\mathrm{d}t + C = -\frac{1}{3}\,(4-2\,t)^{3/2} + C \qquad (19)$$

19. Evaluate integral of exp(3*x - 2) dx

$$y1(x) := \mathrm{e}^{3\,x-2}$$

$$Answer := \int \mathrm{e}^{3\,x-2}\,\mathrm{d}x$$

$$answer := \frac{1}{3}\,\mathrm{e}^{3\,x-2} + C$$

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Creating problem #8

Applying substitution x = 1/3*u+2/3, u = 3*x-2 with dx = 1/3*
du, du = 3*dx
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$$\int \mathrm{e}^{3\,x-2}\,\mathrm{d}x = \int \frac{1}{3}\,\mathrm{e}^{u}\,\mathrm{d}u$$

```
Reverting substitution using u = 3*x-2
```

$$answer := \int \mathrm{e}^{3\,x-2}\,\mathrm{d}x + C = \frac{1}{3}\,\mathrm{e}^{3\,x-2} + C \qquad (20)$$

20. Evaluate integral of 1/(4*x)

For x< 0 get (1/4) ln(|x|) + C

$$y1(x) := \frac{1}{4\,x}$$

$$Answer := \int \frac{1}{4\,x}\,\mathrm{d}x$$

$$answer := \frac{1}{4}\,\ln(x) + C \qquad (21)$$

21. In a certain section of the country, the price of chicken is currently $3 per kilogram. It is estimated that x weeks from now the price will be increasing at a rate of 3*sqrt(t + 1) cents per kilogram, per week. How much will chicken cost 5 weeks from now?

A) $3.27
B) $0.28
C) $4.27
D) $2.28

Final Price = Initial Price + Change; NOTICE cent =.01$

$$y1(t) := 0.03\,\sqrt{t+1}$$

$$Answer := 3.00 + \int_0^5 0.03\sqrt{t+1}\,dt$$

$$answer := 3.273938769 \quad (22)$$

22. Water flows into a tank at the rate of sqrt(8*t + 9) ft^3 /min. If the tank is empty when t = 0, how much water does it contain 8 minutes later? Express the answer to two decimal places.
A) 0.46
B) 49.73
C) 404.71
D) 68.35

Same type problem as above;(21). Initial value at t =0 is 0 (empty)

$$y1(t) := \sqrt{8t+9}$$

$$Answer := \int_0^{8.} \sqrt{8t+9}\,dt$$

$$answer := 49.72602278 \quad (23)$$

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23. Evaluate integral of x*sqrt(x^2 + 9)

Substitution u = x^2 + 9 gives du = 2x dx
Maple code is
Rule[change,u=x^2+9](Int(y1(x),x));
answer:= value(%) + `C`;

$$y1(x) := x\sqrt{x^2+9}$$

$$Problem := \int x\sqrt{x^2+9}\,dx$$

$$answer := \frac{1}{3}\left(x^2+9\right)^{3/2} + C$$

```
Creating problem #10

Applying substitution x = (u-9)^(1/2), u = x^2+9 with dx = 1/2/
(u-9)^(1/2)*du, du = 2*x*dx
```

$$\int x\sqrt{x^2+9}\,dx = \int \frac{1}{2}\sqrt{u}\,du$$

```
Reverting substitution using u = x^2+9
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$$answer := \int x\sqrt{x^2+9}\,dx + C = \frac{1}{3}\left(x^2+9\right)^{3/2} + C \quad (24)$$

24. Evaluate definite integral of (3*x -5)^4
. Express your answer as a decimal. Approximate to one decimal place.
A) 2,250.2
B) 2,251.6
C) 2,252.8
D) 2,253.4

Simple substitution u =3x -5

$$y1(x) := (3x-5)^4$$

$$Problem := \int (3x-5)^4 \, dx$$

$$answer := \frac{1}{15}(3x-5)^5 + C$$

$$FTCanswer := 2252.800000$$

$$numerical\_answer := 2252.800000 \quad (25)$$

25. Use the fundamental theorem of calculus to find the area of the region under the line y = 6x + 9 above the interval $1 \le x \le 4$.
A) 96
B) 90
C) 72
D) 70

Might graph first!

$$y1(x) := 6x+9$$

$$Problem := \int_1^4 (6x+9) \, dx$$

$$answer := 72 \quad (26)$$

26. Suppose the marginal cost is C(x) = exp(-0.9*x)
, where x is measured in units of 200 items
and the cost is measured in units of $6,000. Find the cost corresponding to the production interval [600, 800] .
A) $239
B) $215
C) $266
D) $210

Carefulon units!

$$y1(x) := e^{-0.9x}$$

$$Problem := 6000 \int_3^4 e^{-0.9x} \, dx$$

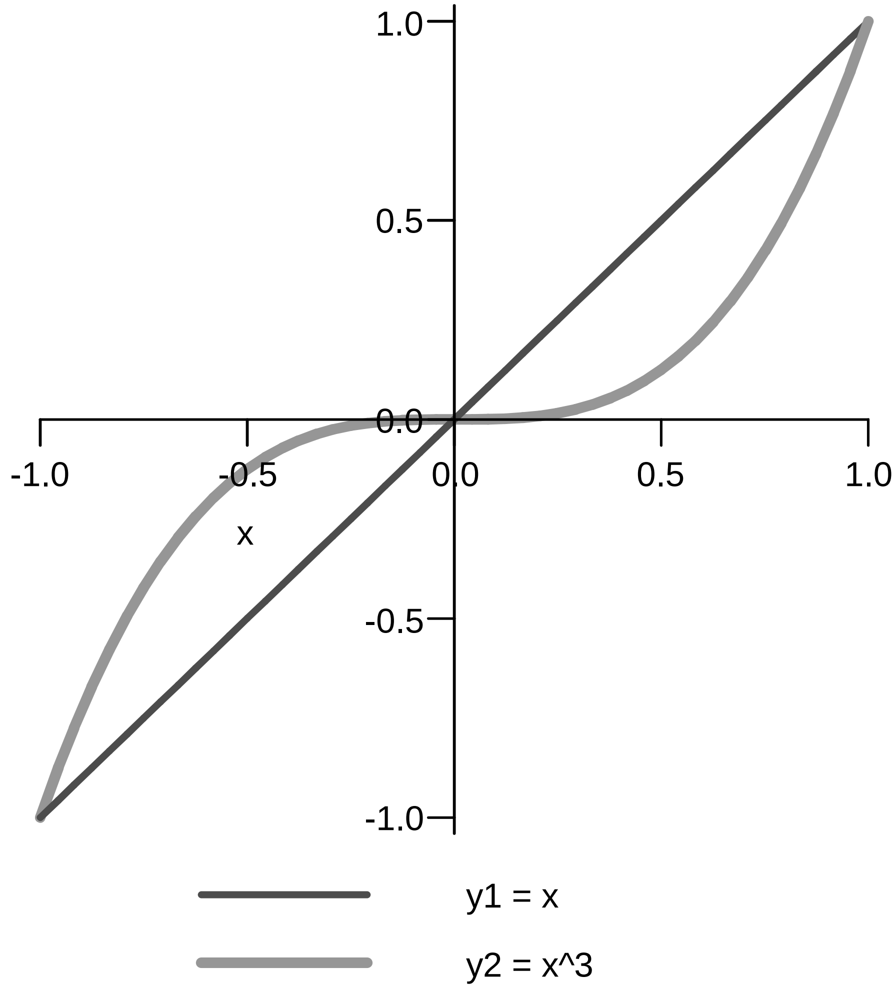
$$answer := 265.8786019 \quad (27)$$

27. Determine the area of the region bounded by the line y = x and the curve y2 = x^3
Two functions - determine region by drawing a graph. Actually two regions! NOT on Test Three!

$$y1(x) := x$$

$$y2(x) := x^3$$

$$intersection\ at\ x = \ := 0, 1, -1$$



$$Area\_Right\_Region := \int_0^1 (x - x^3)\, dx$$

$$area\_right := \frac{1}{4}$$

$$Area\_Left\_Region := \int_{-1}^0 (x^3 - x)\, dx$$

$$area\_left := \frac{1}{4}$$

$$total\_area := \frac{1}{2} \quad (28)$$

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28. Determine the area between y = sqrt(x) and y = x^3 on the domain determined by the points where the graphs of the functions cross..

A) 0.4355
B) 0.4167
C) 0.5563

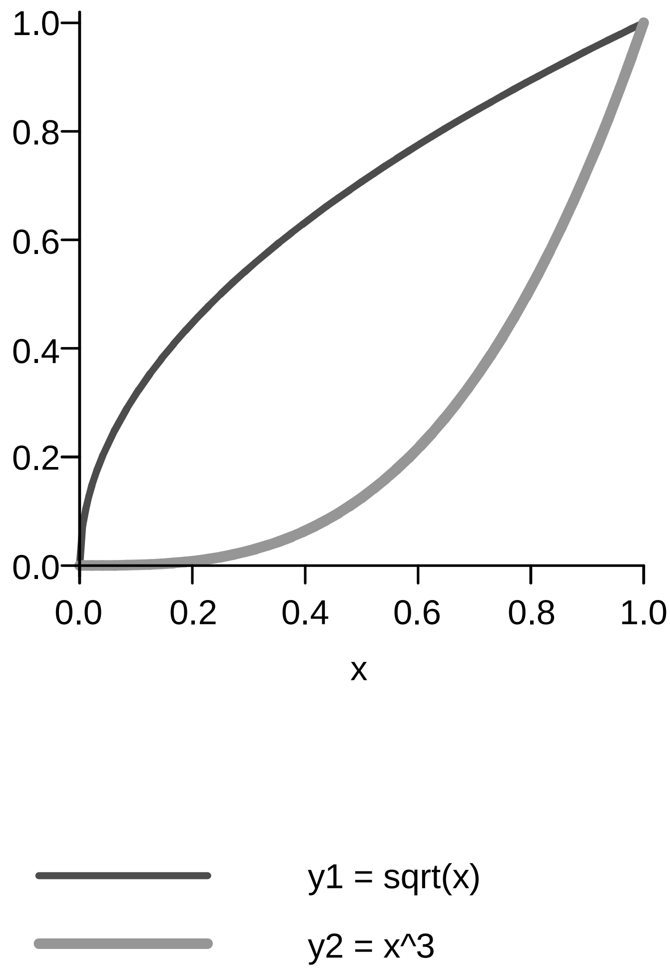
D) 0.7210

FInd intersections (sqrt(x) =x^3) and graph

$$y1(x) := \sqrt{x}$$

$$y2(x) := x^3$$

$$intersection\ at := \{x = 1\}, \{x = 0\}$$



$$Area := \int_0^1 \left(\sqrt{x} - x^3\right) dx$$

$$area := \frac{5}{12}$$

$$area := 0.4167 \qquad (29)$$

29. Sketch the region R and then use calculus to find the area of R. R is the region between the curve y = x^3 and the line y = 20x for x ≥ 0 .

A) 0

B) 100

C) 5

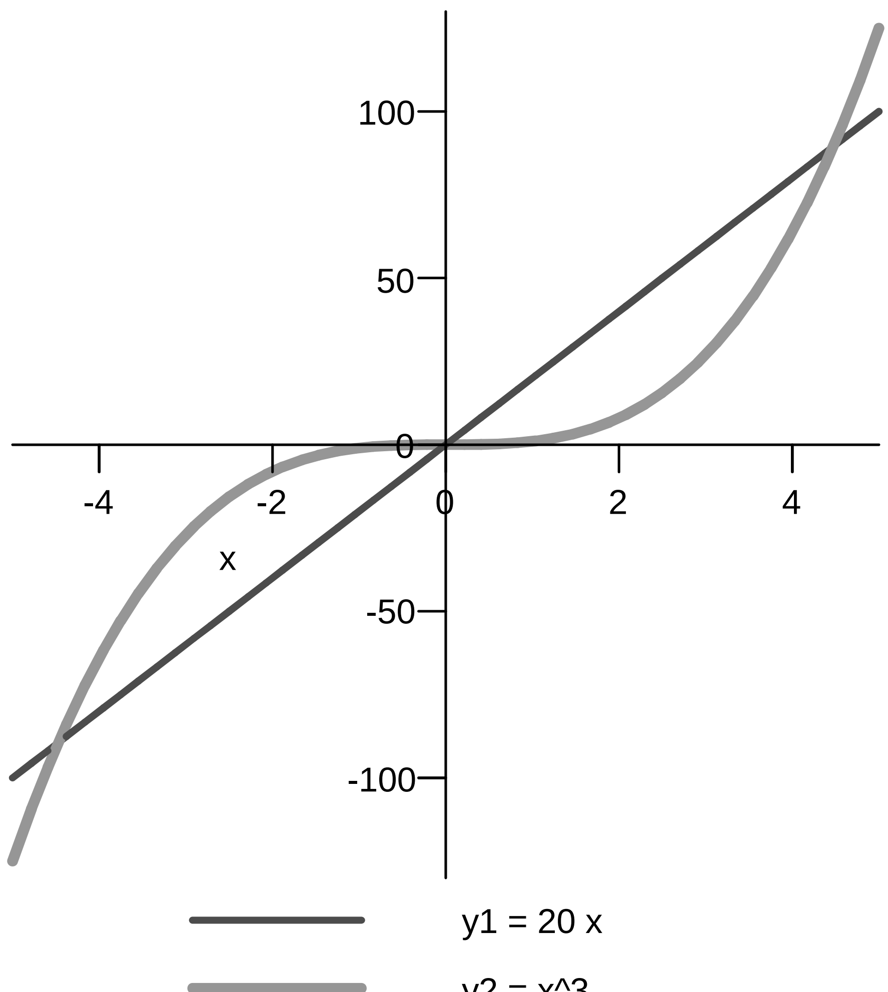
D) 25

Again twoareas but only ONE if you notice x >= 0

$$y1(x) := 20\,x$$

$$y2(x) := x^3$$

$$intersection\ at\ x = := 0,\ 2\sqrt{5},\ -2\sqrt{5}$$



$$Right\_Area := \int_0^{2\sqrt{5}} \left(20\,x - x^3\right)\,dx$$

$$right\_area := 100$$

$$area := 100. \qquad (30)$$

30. Find the consumers surplus for a commodity whose demand function is
D(q) = 30 * exp(-0.03*q)
dollars per unit if the market price is p_0$ =21 dollars per unit. (Hint: Find the quantity q0 that corresponds to the given price p0 = D(q0).)
A) $49.53
B) $49.81
C) $50.33
D) $53.41

Need q_0 solve demand =$21

$$Demand(q) := 30\, e^{-0.03\, q}$$

$$p\_0 := 21$$

$$q\_0 := 11.88916480$$

$$CS(q\_0) := \int_0^{11.88916480} 30\, e^{-0.03\, q}\, dq - 249.6724608$$

$$answer := 50.3275392 \qquad (31)$$

31. Money is transferred continuously into an account at the constant rate of $1,400 per year. The account earns interest at the annual rate of 7% compounded continuously. How much will be in the account at the end of 2 years?
A) $2,299.55
B) $81,103
C) $23,004.48
D) $2,800

NO CORRECT ABSWER GIVEN

$$y1(t) := 1400\, e^{0.14 - 0.07\, t}$$

$$Answer := \int_0^{2.} 1400\, e^{0.14 - 0.07\, t}\, dt$$

$$answer := 3005.475977 \qquad (32)$$

32. It is estimated that t days from now a farmer's crop will be increasing at the rate of 0.3 * t^2 + 0.6*t + 1 bushels per day. By how much will the value of the crop increase during the next 7 days if the market price remains fixed at $2 per bushel?
A) $98.00
B) $112.00
C) $122.00
D) $28.00

Total Change is wanted

$$y1(t) := 0.3\, t^2 + 0.6\, t + 1$$

$$Answer := 2 \int_0^{7} \left(0.3\, t^2 + 0.6\, t + 1\right) dt$$

$$answer := 112. \qquad (33)$$

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33. Money is transferred continuously into an account at the constant rate of $1,200 per year. Assume the account earns interest at the annual rate of 3% compounded continuously. Compute the future value of the income stream over a 11 year period.
A) $469.16
B) $31,277.45
C) $62,554.9
D) $15,638.73

Future Value

$$y1(t) := 1200\, e^{0.33 - 0.03\, t}$$

$$Answer := \int_0^{11} 1200\, e^{0.33 - 0.03\, t}\, dt$$

$$answer := 15638.72514 \tag{34}$$