math165sec_3_4.mw
Maple 10 Worksheet for Special Assignment One 2006 (165sa106.tex) in Math 165 - Calculus for Business.

First load plots and student:

```
> restart: with( student):with (plots):
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MAXIMUM PROFIT AND MINIMUM AVERAGE COST, Prob. 17-22

Given a price function p(q) at which q units of a particular quantity are sold and the total cost C(q) of producing q uniits,

(a) Find the profit function P(q) = R(q) - C(q) = q*p(q) - C(q), the maginal revenue R'(q) and the marginal cost C'(q).
Sketch the graphs of P(q), R'(q), and C'(q) on the same coordinate axes and determine the level of produvtion q
where P(q) is maximized.

(b) Find the average cost and sketch the graphs of A(q) and the marginal cost C'(q) on the same coordinate axes.
Determine the level of production q where A(q) is minimized.

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Problem 17

```
> p_17(q):= 49 - q;
  C_17(q):= (1/8)*q^2 + 4*q + 200;
  R_17(q):= q*p_17(q);
  P_17(q):= R_17(q) - C_17(q);
```

$$p\_17(q) := 49 - q$$

$$C\_17(q) := \frac{1}{8} q^2 + 4 q + 200$$

$$R\_17(q) := q (49 - q)$$

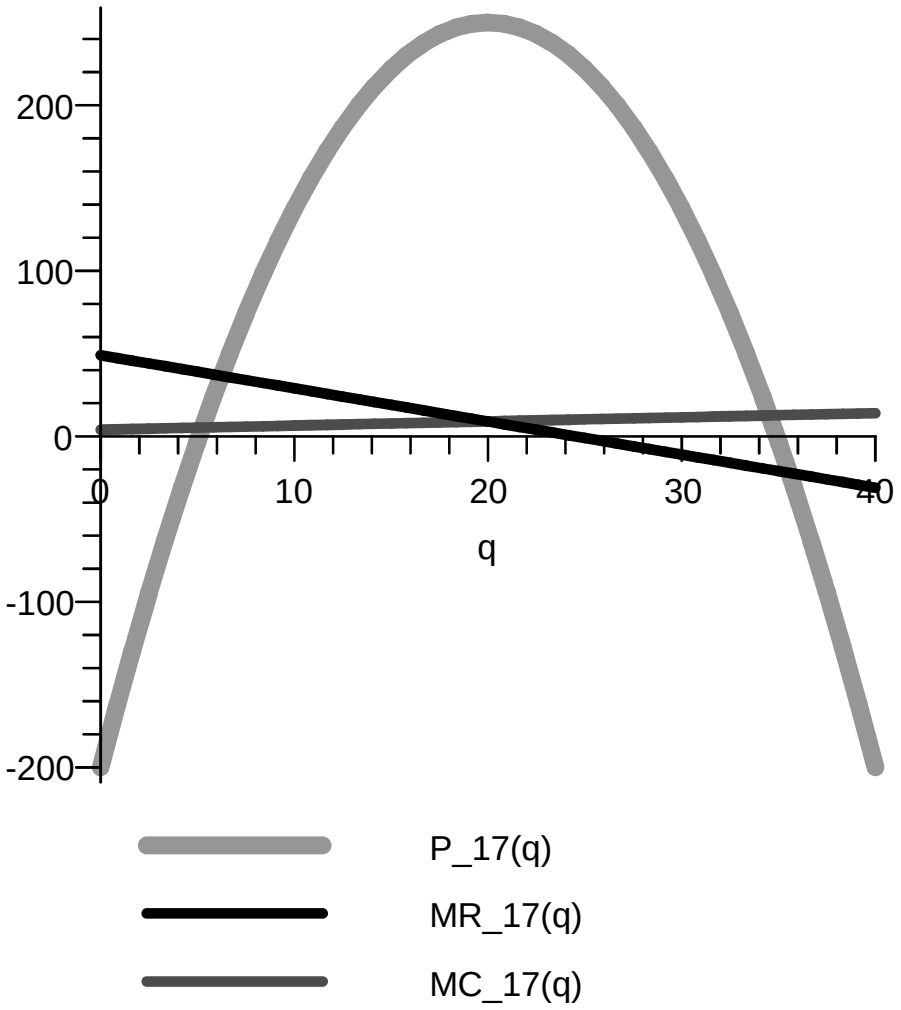
$$P\_17(q) := q (49 - q) - \frac{1}{8} q^2 - 4 q - 200 \quad (1)$$

```
> MR_17(q):=diff(R_17(q),q);
  MC_17(q):=diff(C_17(q),q);
```

$$MR\_17(q) := 49 - 2 q$$

$$MC\_17(q) := \frac{1}{4} q + 4 \quad (2)$$

```
> plot([P_17(q),MR_17(q),MC_17(q)],q=0..40,color=[green,black,red],
  thickness=[5,3,3],legend=[`P_17(q)`,`MR_17(q)`,`MC_17(q)`]);
```




```
> solve(MR_17(q)=MC_17(q),q);
  maximize(P_17(q),q,location);
```

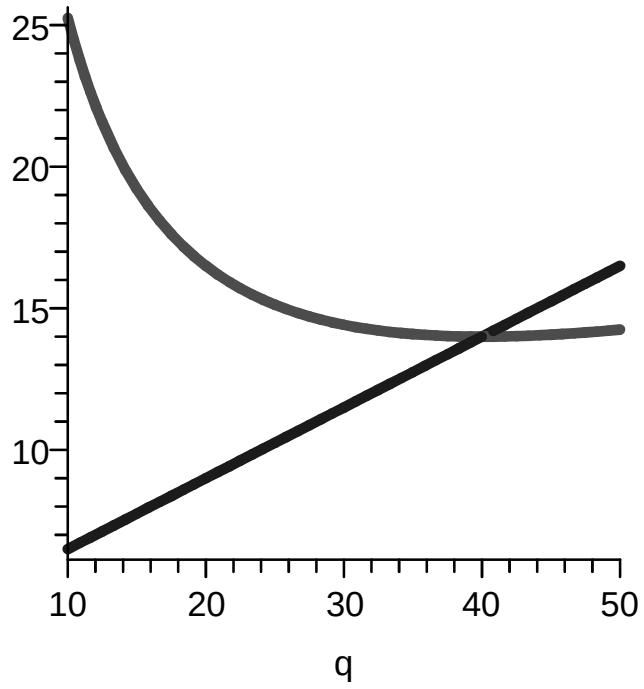
$$20$$

$$250,\ \{[\{q=20\},250]\} \tag{3}$$

```
> A_17(q):=C_17(q)/q;
```

$$A\_17(q) := \frac{\frac{1}{8}\,q^2+4\,q+200}{q} \tag{4}$$

```
> plot([A_17(q),MC_17(q)],q=10..50,thickness=[3,3],color=[red,blue],
  legend=[`A_17(q)`,`MC_17(q)`]);
```




```
> minimize(A_17(q),q=10..50,location);
  solve({A_17(q)=MC_17(q),q >0},q);
```

$$14, \{[\{q = 40\}, 14]\}$$

$$\{q = 40\} \tag{5}$$

****

Problem 18

```
> p_18(q):= 37- 2*q;
  C_18(q):= 3*q^2 + 5*q + 75;
  R_18(q):= q*p_18(q);
  P_18(q):= R_18(q) - C_18(q);simplify(%);
```

$$p\_18(q) := 37 - 2\,q$$

$$C\_18(q) := 3\,q^2 + 5\,q + 75$$

$$R\_18(q) := q\,(37 - 2\,q)$$

$$P\_18(q) := q\,(37 - 2\,q) - 3\,q^2 - 5\,q - 75$$

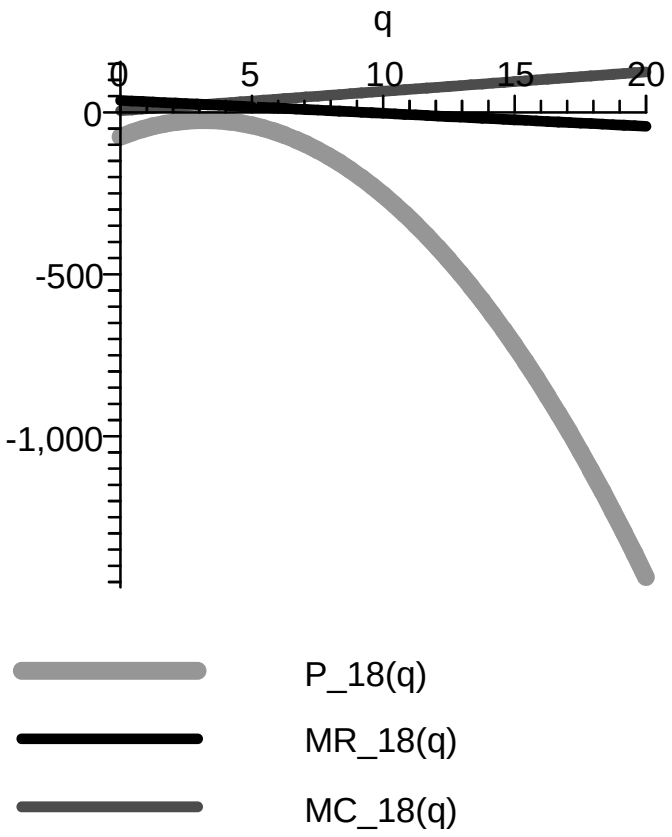
$$32\,q - 5\,q^2 - 75 \tag{6}$$

```
> MR_18(q):=diff(R_18(q),q);
  MC_18(q):=diff(C_18(q),q);
```

$$MR\_18(q) := 37 - 4\,q$$

$$MC\_18(q) := 6\,q + 5 \tag{7}$$

```
> plot([P_18(q),MR_18(q),MC_18(q)],q=0..20,color=[green,black,red],
  thickness=[5,3,3],legend=[`P_18(q)`,`MR_18(q)`,`MC_18(q)`]);
```




```
> solve(MR_18(q)=MC_18(q),q);
  maximize(P_18(q),q,location);
```

$$\frac{16}{5}$$

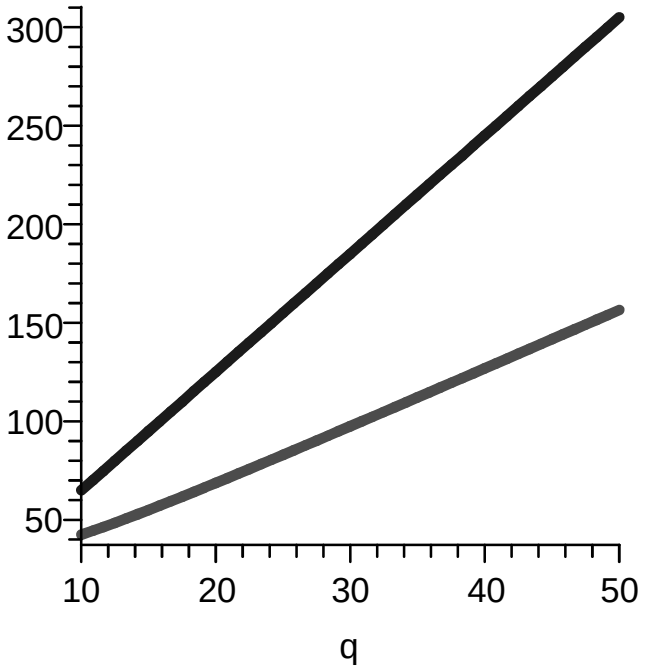
$$-\frac{119}{5},\ \left\{\left[\left\{q = \frac{16}{5}\right\}, -\frac{119}{5}\right]\right\} \tag{8}$$

```
> A_18(q):=C_18(q)/q;
```

$$A\_18(q) := \frac{3\,q^2 + 5\,q + 75}{q} \tag{9}$$

```
> plot([A_18(q),MC_18(q)],q=10..50,thickness=[3,3],color=[red,blue],
```

```
   legend=[`A_18(q)`,`MC_18(q)`]);
```




```
> minimize(A_18(q),q=10..50,location);
   solve({A_18(q)=MC_18(q),q >0},q);
```

$$\frac{85}{2},\ \left\{\left[\{q=10\},\ \frac{85}{2}\right]\right\}$$

$$\{q=5\} \tag{10}$$

```
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```

Problem 19

```
> p_19(q):= 180 - 2*q;
   C_19(q):= q^3 + 5*q +162;
   R_19(q):= q*p_19(q);
   P_19(q):= R_19(q) - C_19(q);simplify(%);
```

$$p\_19(q) := 180 - 2\,q$$

$$C\_19(q) := q^3 + 5\,q + 162$$

$$R\_19(q) := q\,(180 - 2\,q)$$

$$P\_19(q) := q\,(180 - 2\,q) - q^3 - 5\,q - 162$$

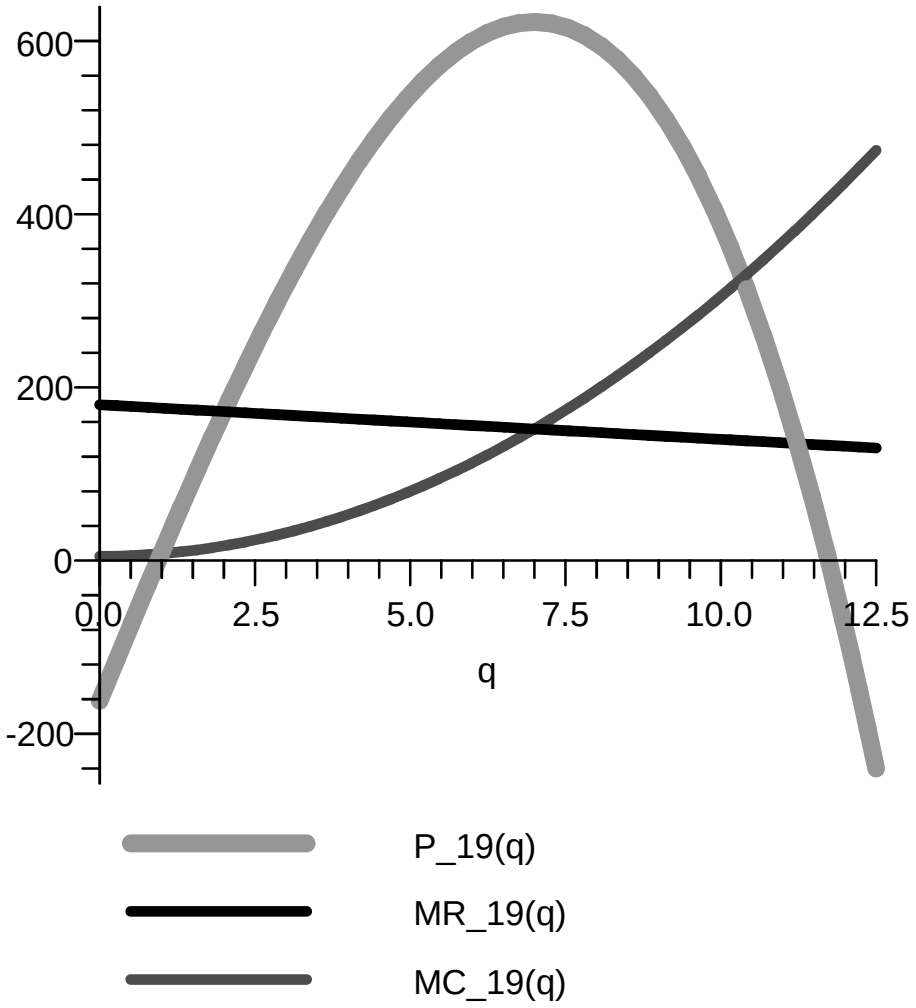
$$175\,q - 2\,q^2 - q^3 - 162 \tag{11}$$

```
> MR_19(q):=diff(R_19(q),q);
  MC_19(q):=diff(C_19(q),q);
```

$$MR\_19(q) := 180 - 4\,q$$

$$MC\_19(q) := 3\,q^2 + 5 \tag{12}$$

```
> plot([P_19(q),MR_19(q),MC_19(q)],q=0.. 12.5,color=[green,black,
  red],thickness=[5,3,3],legend=[`P_19(q)`,`MR_19(q)`,`MC_19(q)`]);
```




```
> solve([MR_19(q)=MC_19(q),q>0],q);
  maximize(P_19(q),q=0 .. 12.5,location);
```

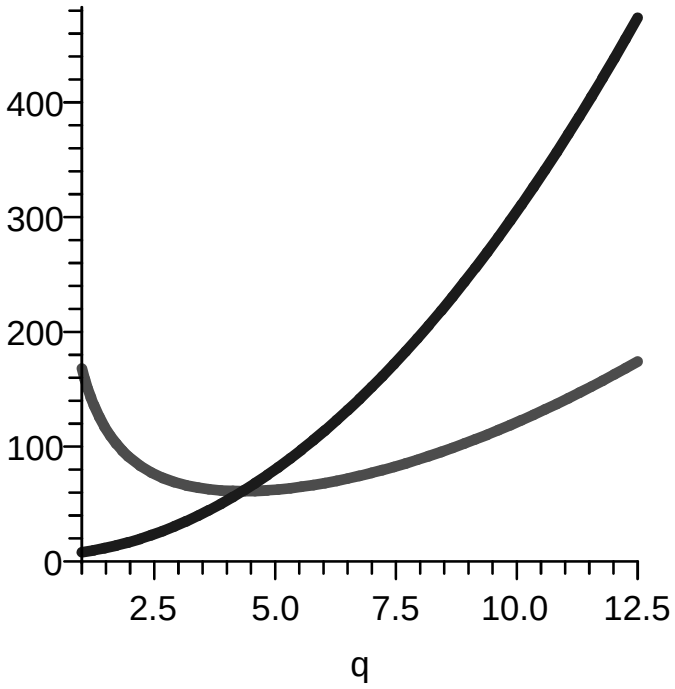
$$\{q = 7\}$$

$$622,\ \{[\{q = 7\}, 622]\} \tag{13}$$

```
> A_19(q):=C_19(q)/q;
```

$$A\_19(q) := \frac{q^3 + 5\,q + 162}{q} \tag{14}$$

```
> plot([A_19(q),MC_19(q)],q=1..12.5,thickness=[3,3],color=[red,blue]
  ,legend=[`A_19(q)`,`MC_19(q)`]);
```




```
> minimize(A_19(q),q=10..50,location);
  solve([A_19(q)=MC_19(q),q >0, q <= 12.5],q);
```

$$\frac{606}{5}, \left\{ \left[\{q = 10\}, \frac{606}{5} \right] \right\}$$

$$\{q = 4.326748711\} \tag{15}$$

****

Problem 20

```
> p_20(q):= 710 - 1.1*q^2;
  C_20(q):= 2*q^3 - 23*q^2 + 90.7*q +151;
  R_20(q):= q*p_20(q);
  P_20(q):= R_20(q) - C_20(q);simplify(%);
```

$$p\_20(q) := 710 - 1.1\,q^2$$

$$C\_20(q) := 2\,q^3 - 23\,q^2 + 90.7\,q + 151$$

$$R\_20(q) := q\left(710 - 1.1\,q^2\right)$$

$$P\_20(q) := q\left(710 - 1.1\,q^2\right) - 2\,q^3 + 23\,q^2 - 90.7\,q - 151$$

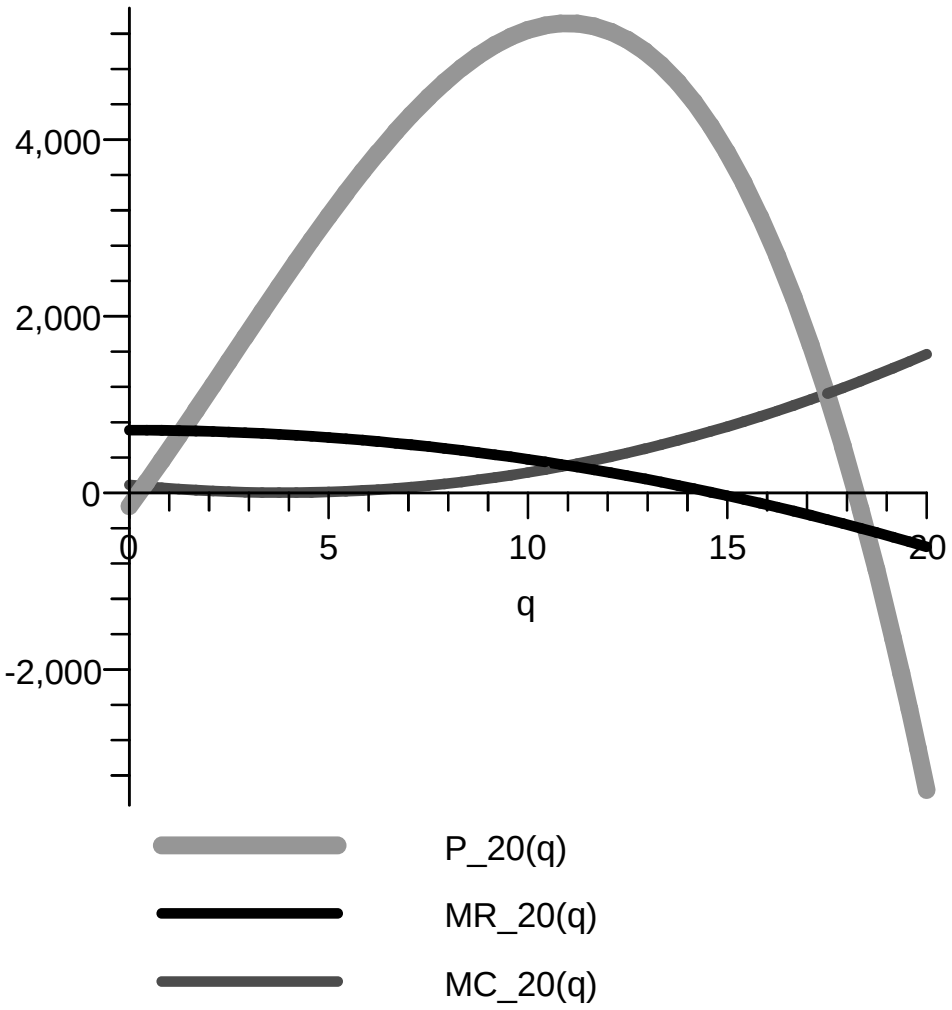
$$619.3000000\,q - 3.100000000\,q^3 + 23.\,q^2 - 151. \tag{16}$$

```
> MR_20(q):=diff(R_20(q),q);
  MC_20(q):=diff(C_20(q),q);
```

$$MR\_20(q) := 710. - 3.3\,q^2$$

$$MC\_20(q) := 6\,q^2 - 46\,q + 90.7 \tag{17}$$

```
> plot([P_20(q),MR_20(q),MC_20(q)],q=0.. 20,color=[green,black,red],
  thickness=[5,3,3],legend=[`P_20(q)`,`MR_20(q)`,`MC_20(q)`]);
```




```
> solve([MR_20(q)=MC_20(q),q>0],q);
  maximize(P_20(q),q=0 .. 12.5,location);
```

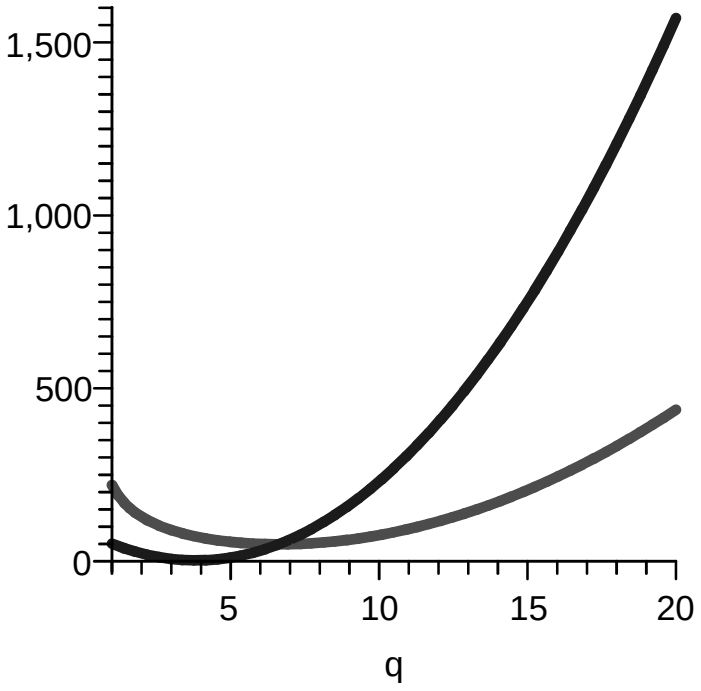
$$\{q = 11.\}$$

$$5318.2,\ \{[\{q = 11.\}, 5318.2]\} \tag{18}$$

```
> A_20(q):=C_20(q)/q;
```

$$A\_20(q) := \frac{2\,q^3 - 23\,q^2 + 90.7\,q + 151}{q} \tag{19}$$

```
> plot([A_20(q),MC_20(q)],q=1..20,thickness=[3,3],color=[red,blue],
  legend=[`A_20(q)`,`MC_20(q)`]);
```




```
>
  solve([diff(A_20(q),q)=0,q>0],q);
  minimize(A_20(q),q=0..20,location);
  solve([A_20(q)=MC_20(q),q >0, q <= 20],q);
```

$$\{q = 6.613171961\}$$

$$48.89835011, \{[\{q = 6.613171961\}, 48.89835011]\}$$

$$\{q = 6.613171961\} \qquad (20)$$

```
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```

Problem 21

```
> p_21(q):= 1.0625 - 0.0025*q;
  C_21(q):= (q^2 + 1)/(q + 3);
  R_21(q):= q*p_21(q);
  P_21(q):= R_21(q) - C_21(q);simplify(%);
```

$$p\_21(q) := 1.0625 - 0.0025\,q$$

$$C\_21(q) := \frac{q^2+1}{q+3}$$

$$R\_21(q) := q\,(1.0625 - 0.0025\,q)$$

$$P\_21(q) := q\,(1.0625 - 0.0025\,q) - \frac{q^2+1}{q+3}$$

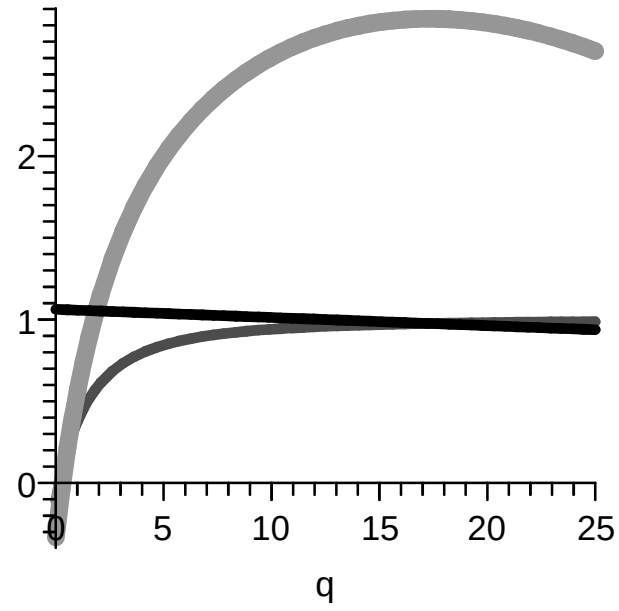
$$-\frac{0.002500000000\,\left(-22.\,q^2 - 1275.\,q + q^3 + 400.\right)}{q+3.} \tag{21}$$

```
> MR_21(q):=diff(R_21(q),q);
  MC_21(q):=diff(C_21(q),q);
```

$$MR\_21(q) := 1.0625 - 0.0050\,q$$

$$MC\_21(q) := \frac{2\,q}{q+3} - \frac{q^2+1}{(q+3)^2} \tag{22}$$

```
> plot([P_21(q),MR_21(q),MC_21(q)],q=0.. 25,color=[green,black,red],
  thickness=[5,3,3],legend=[`P_21(q)`,`MR_21(q)`,`MC_21(q)`]);
```



```
> solve([MR_21(q)=MC_21(q),q>0],q);
  maximize(P_21(q),q=0 .. 20,location);
```

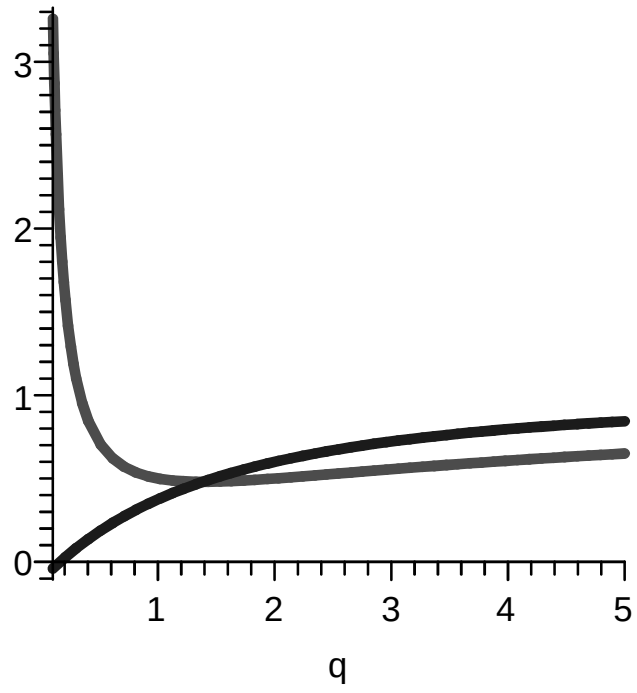
$$\{q = 17.33609536\}$$

$$2.84041898,\ \{[\{q = 17.33609536\}, 2.84041898]\} \tag{23}$$

```
> A_21(q):=C_21(q)/q;
```

$$A\_21(q) := \frac{q^2 + 1}{(q + 3)\, q} \tag{24}$$

```
> plot([A_21(q),MC_21(q)],q=0.1 .. 5,thickness=[3,3],color=[red,
  blue],legend=[`A_21(q)`,`MC_21(q)`]);
```




```
> solve([diff(A_21(q),q)=0,q>0,q < 6],q);
  minimize(A_21(q),q=0 .. 5,location);
  solve([A_21(q)=MC_21(q),q >0, q <= 20],q);
```

$$\{q = \mathrm{RootOf}(3\,\_Z^2 - 2\,\_Z - 3,\ index = 1)\}$$

$$\frac{\left(\frac{1}{3}+\frac{1}{3}\sqrt{10}\right)^2+1}{\left(\frac{10}{3}+\frac{1}{3}\sqrt{10}\right)\left(\frac{1}{3}+\frac{1}{3}\sqrt{10}\right)},\left\{\left[\left\{q=\frac{1}{3}+\frac{1}{3}\sqrt{10}\right\},\ \frac{\left(\frac{1}{3}+\frac{1}{3}\sqrt{10}\right)^2+1}{\left(\frac{10}{3}+\frac{1}{3}\sqrt{10}\right)\left(\frac{1}{3}+\frac{1}{3}\sqrt{10}\right)}\right]\right\}$$

$$\left\{q=\text{RootOf}\left(3\_Z^2-2\_Z-3,\ index=1\right)\right\} \tag{25}$$

****

Problem 22

```
> p_22(q):= 81 - 3*q;
  C_22(q):= (q + 1)/(q + 3);
  R_22(q):= q*p_22(q);
  P_22(q):= R_22(q) - C_22(q);simplify(%);
```

$$p\_22(q) := 81 - 3\,q$$

$$C\_22(q) := \frac{q+1}{q+3}$$

$$R\_22(q) := q\,(81 - 3\,q)$$

$$P\_22(q) := q\,(81 - 3\,q) - \frac{q+1}{q+3}$$

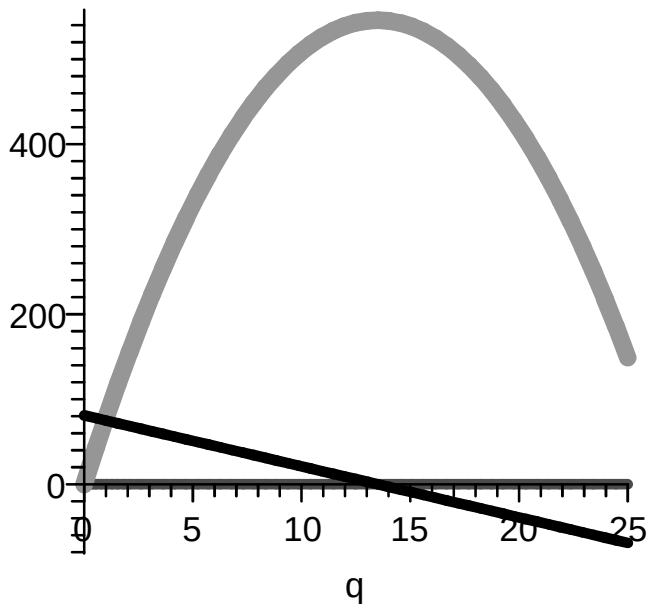
$$-\frac{-72\,q^2 - 242\,q + 3\,q^3 + 1}{q+3} \tag{26}$$

```
> MR_22(q):=diff(R_22(q),q);
  MC_22(q):=diff(C_22(q),q);
```

$$MR\_22(q) := 81 - 6\,q$$

$$MC\_22(q) := \frac{1}{q+3} - \frac{q+1}{(q+3)^2} \tag{27}$$

```
> plot([P_22(q),MR_22(q),MC_22(q)],q=0.. 25,color=[green,black,red],
  thickness=[5,3,3],legend=[`P_22(q)`,`MR_22(q)`,`MC_22(q)`]);
```




```
> solve([MR_22(q)=MC_22(q),q>0],q);
  maximize(P_22(q),q=0 .. 20,location);
```

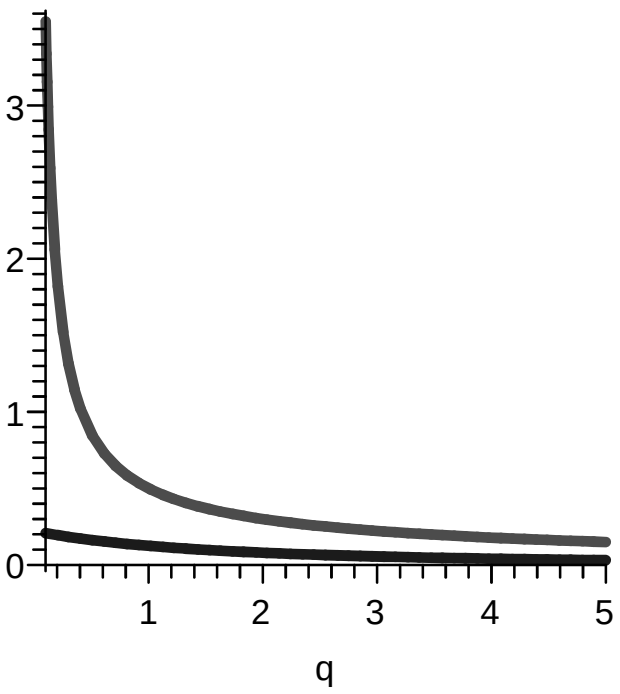
$$\left\{q = \text{RootOf}\left(-45\,\_Z^2 - 432\,\_Z - 727 + 6\,\_Z^3,\ index = 1\right)\right\}$$

$$maximize\left(q\,(81 - 3\,q) - \frac{q+1}{q+3},\ q = 0..20,\ location\right),\ \{\} \tag{28}$$

```
> A_22(q):=C_22(q)/q;
```

$$A\_22(q) := \frac{q+1}{(q+3)\,q} \tag{29}$$

```
> plot([A_22(q),MC_22(q)],q=0.1 .. 5,thickness=[3,3],color=[red,
  blue],legend=[`A_22(q)`,`MC_22(q)`]);
```




```
> solve([diff(A_22(q),q)=0,q>0,q < 6],q);
  minimize(A_22(q),q=0 .. 5,location);
  solve([A_22(q)=MC_22(q),q >0, q <= 20],q);
```

$$\frac{3}{20}, \left\{ \left[\{q = 5\}, \frac{3}{20} \right] \right\}$$

```
Warning, solutions may have been lost
>
>
>
```