math165_3_4_elasto.mw
Maple 10 Worksheet for Elasticity in Math 165 - Calculus for Business.

First load plots and student:

```
> restart:with( student):with (plots):
```

If a quantity x is changed by an amount Δx, the {\it
relative change in x\/} is the ratio Δx/x. The {\it
percentage change in x is   100 Δx/x. Note that there
are {\it no units\/} for the ratio of two quantities with the same
units -- although the ratio might be expressed in percent. In this context, one says
"percentage change in x."

The discussion tacitly assumes that percentage change in x is on the order
of a few percent.

Suppose the quantity q and the price p are related, e.g., by a
relation of the form q = D(p).

Then the p elasticity of q is defined as the
(Δq/q)/(Δp/p),
roughly,
(Percentage change in q)/(Percentage change in p)

Write the p elasticity of q as
(Δq/q)/(Δp/p) = (p/q)(Δq/Δp).,
after a small manipulation.

Taking the limit as Δp → 0, we define

the "p" elasticity of q" as

E(p) = (p/q)*(dq/dp)
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Problems Section 3.4 p. 250 Hoffmann 9e
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Problems 23--28

For the given demand function q = D(p), compute the [price] elasticity
of demand and determine whether the demand is elastic, inelastic, or of unit elasticity.

We use the notation:
D = D(p), demand function - demand as a function of price
ddp_D = D'(p), derivative of demand wrt p
E = E(p), price elasticity of demand
R = p q = p*D(p), Revenue
MR = Marginal Revenue wrt price (some abuse of notation since in other contexts
marginal revenue is the derivative wrt demand (quantity))
Sign of MR determines whether R increasing/decreasing wrt price.
p_0 = the value of p at for which E(p) is found.

N.B. To create functions we use the arrow (equivalent to defining a procedure) so that f:= x -> x^2; is to be read as `f maps x to x^2` and f(3):= 3^2

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Problem 23

```
> D_23:=p -> - 1.3*p + 10; p_0:=4;
  ddpD_23(p):= diff(D_23(p),p);
  E_23:=p -> (p/D_23(p))*(ddpD_23(p));`E_23(p)`:=E_23(p);
  R_23:=p ->  p * D_23(p);
  MR_23:= p -> diff(R_23(p),p);`MR_23(p)  `:=MR_23(p);
  E_23(p_0):=eval(E_23(p),p=p_0);
  MR(4):=eval(MR_23(p),p=p_0);
```

$$D\_23 := p \to -1.3\,p + 10$$

$$p\_0 := 4$$

$$ddpD\_23(p) := -1.3$$

$$E\_23 := p \to \frac{p\; ddpD\_23(p)}{D\_23(p)}$$

$$E\_23(p) := -\frac{1.3\,p}{-1.3\,p + 10}$$

$$R\_23 := p \to p\; D\_23(p)$$

$$MR\_23 := p \to \frac{d}{dp}\, R\_23(p)$$

$$MR\_23(p) := -2.6\,p + 10.$$

$$E\_23(4) := -1.083333333$$

$$MR(4) := -.4 \qquad (1)$$

****

Problem 24

```
> D_24(p):= - 1.5*p + 25; p_0:= 12;
  ddpD_24(p):= diff(D_24(p),p):`ddp_24(p)  `:=ddpD_24(p);
  E_24(p):= (p/D_24(p))*(ddpD_24(p));
  R_24(p):= p*D_24(p);
  MR_24(p):=diff(R_24(p),p);
  E(p_0):=eval(E_24(p),p=p_0);
  MR(p_0):=eval(MR_24(p),p=p_0);
```

$$D\_24(p) := -1.5\,p + 25$$

$$p\_0 := 12$$

$$ddp\_24(p) := -1.5$$

$$E\_24(p) := -\frac{1.5\,p}{-1.5\,p + 25}$$

$$R\_24(p) := p\,(-1.5\,p + 25)$$

$$MR\_24(p) := -3.0\,p + 25.$$

$$E(12) := -2.571428571$$

$$MR(12) := -11.0 \tag{2}$$

****

Problem 25

```
> D_25(p):= 200 - p^2; p_0:= 10;
  ddpD_25(p):= diff(D_25(p),p);
  E_25(p):= (p/D_25(p))*(ddpD_25(p));
  R_25(p):= p*D_25(p);
  MR_25(p):=diff(R_25(p),p);
  E(p_0):=eval(E_25(p),p=10);
  MR(p_0):=eval(MR_25(p),p=10);
```

$$D\_25(p) := 200 - p^2$$

$$p\_0 := 10$$

$$ddpD\_25(p) := -2\,p$$

$$E\_25(p) := -\frac{2\,p^2}{200 - p^2}$$

$$R\_25(p) := p\,\left(200 - p^2\right)$$

$$MR\_25(p) := 200 - 3\,p^2$$

$$E(10) := -2$$

$$MR(10) := -100 \tag{3}$$

****

Problem 26

```
> D_26(p):= sqrt(400 - 0.01*p^2);p_0:=120;
  ddpD_26(p):= diff(D_26(p),p);
  E_26(p):= (p/D_26(p))*(ddpD_26(p));
  R_26(p):= p*D_26(p);
  MR_26(p):=diff(R_26(p),p);
  E(p_0):=eval(E_26(p),p=120);
  MR(p_0):=eval(MR_26(p),p=120);
```

$$D\_26(p) := \sqrt{400 - 0.01\,p^2}$$

$$p\_0 := 120$$

$$ddpD\_26(p) := -\frac{0.01000000000\,p}{\sqrt{400 - 0.01\,p^2}}$$

$$E\_26(p) := -\frac{0.01000000000\,p^2}{400 - 0.01\,p^2}$$

$$R\_26(p) := p\,\sqrt{400 - 0.01\,p^2}$$

$$MR\_26(p) := \sqrt{400-0.01\,p^2} - \frac{0.01000000000\,p^2}{\sqrt{400-0.01\,p^2}}$$

$$E(120) := -.5625000000$$

$$MR(120) := 7.000000000 \qquad (4)$$

****

Problem 27

```
> D_27(p):= 3000/p - 100;p_0:=10;
  ddpD_27(p):= diff(D_27(p),p);
  E_27(p):= (p/D_27(p))*(ddpD_27(p));
  normal_E_27(p):=normal(E_27(p));
  R_27(p):= p*D_27(p);
  normal_R_27(p):=normal(R_27(p));
  MR_27(p_0):=diff(R_27(p),p);
  E(p_0):=eval(E_27(p),p=p_0);
  MR(10):=eval(MR_27(p),p=p_0);
```

$$D\_27(p) := \frac{3000}{p} - 100$$

$$p\_0 := 10$$

$$ddpD\_27(p) := -\frac{3000}{p^2}$$

$$E\_27(p) := -\frac{3000}{p\left(\frac{3000}{p} - 100\right)}$$

$$normal\_E\_27(p) := \frac{30}{-30+p}$$

$$R\_27(p) := p\left(\frac{3000}{p} - 100\right)$$

$$normal\_R\_27(p) := 3000 - 100\,p$$

$$MR\_27(10) := -100$$

$$E(10) := -\frac{3}{2}$$

$$MR(10) := -100 \qquad (5)$$

****

Problem 28

```
> D_28(p):= 2000/p^2;p_0:=5;
  ddpD_28(p):= diff(D_28(p),p);
  E_28(p):= (p/D_28(p))*(ddpD_28(p));
  R_28(p):= p*D_28(p);
  MR_28(p):=diff(R_28(p),p);
  E(p_0):=eval(E_28(p),p=p_0);
```

```
MR(10):=eval(MR_28(p),p=p_0);
```

$$D\_28(p) := \frac{2000}{p^2}$$

$$p\_0 := 5$$

$$ddpD\_28(p) := -\frac{4000}{p^3}$$

$$E\_28(p) := -2$$

$$R\_28(p) := \frac{2000}{p}$$

$$MR\_28(p) := -\frac{2000}{p^2}$$

$$E(5) := -2$$

$$MR(10) := -80 \quad (6)$$

****

Problems 39 and 40 calculate elasticity (price elasticity of demand) when the relation between price and demand is given implicitly.

This is an application of implicit differentiation. We use a a Maple command "implicitdiff", but also show the technical details of finding dq/dp in terms of p and q.

Given a particular value of p, finding q (quantity) by "solving the equation" also requires choosing the "positive solution."

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Problem 39 ELASTICITY OF DEMAND

Implicit Relation between p and q.

eqn_39: = Q(p)^2 +* 3*p*Q(p) = 22

```
> eqn_39:= q^2 + 3*p*q -22 = 0;
  dq_dp(p,q):=implicitdiff(eqn_39,q, p);

  ddx_eqn_39:=proc(p);
      diff(q(p)^2 + 3*p*q(p) -22, p);
      convert(%,D);
  end proc:
  first:=ddx_eqn_39(p);
  solve(first,D(q)(p));
```

$$eqn\_39 := q^2 + 3\,p\,q - 22 = 0$$

$$dq\_dp(p, q) := -\frac{3\,q}{2\,q + 3\,p}$$

$$first := 2\,q(p)\,\mathrm{D}(q)(p) + 3\,q(p) + 3\,p\,\mathrm{D}(q)(p)$$

(7)

$$-\frac{3\,q(p)}{2\,q(p)+3\,p} \tag{7}$$

Solve the equation -- we are interested in the positive root

```
> q(p):=solve(eqn_39,q)[1];
> P_39:= 3;Q_39:=eval(q(p),p=P_39);
> E_39:= (P_39/Q_39)*(- 3* Q_39/(2*Q_39 + 3*P_39));`E_39(3) `:=
  simplify(%);
```

$$q(p):=-\frac{3}{2}\,p+\frac{1}{2}\,\sqrt{9\,p^2+88}$$

$$P\_39:=3$$

$$Q\_39:=-\frac{9}{2}+\frac{1}{2}\,\sqrt{169}$$

$$E\_39:=-\frac{9}{169}\,\sqrt{169}$$

$$E\_39(3)\;:=-\frac{9}{13} \tag{8}$$

****

Problem 40 ELASTICITY OF DEMAND
Implicit Relation between p and q.
eqn_40 = Q(p)^2 +* 3*p*Q(p) = 22

```
> restart;eqn_40:= q^2 + 2*p^2 -41 = 0; P_40:= 4;
  `dq/dp`:=implicitdiff(eqn_40,q, p);

  ddx_eqn_40:=proc(p);
      diff(q(p)^2 + 2*p^2 -41, p);
      convert(%,D);
  end proc:
  first:=ddx_eqn_40(p);
  solve(first,D(q)(p));
  q(p):=solve(eqn_40,q)[1];
  Q_40:=eval(q(p),p=P_40);
  E_40:= (P_40/Q_40)*(-2*P_40/(Q_40));
```

$$eqn\_40:=q^2+2\,p^2-41=0$$

$$P\_40:=4$$

$$dq/dp:=-\frac{2\,p}{q}$$

$$first:=2\,q(p)\,\mathrm{D}(q)(p)+4\,p$$

$$-\frac{2\,p}{q(p)}$$

$$q(p):=\sqrt{-2\,p^2+41}$$

$$Q\_40 := \sqrt{9}$$
$$E\_40 := -\frac{32}{9} \tag{9}$$