9e math165_2_4_70.mw
Maple 10 Worksheet for Problems in Math 165 - Calculus for Business.

First load plots and student:

```
> restart:with(student):with (plots):
```

Problem 2.4.70 COMPOUND INTEREST

If \$10000 is invested at an annual rate r (expressed as a decimal) compounded weekly, the total amount
principal P and interest) accumulated after 10 years is given by the formula
A = 10000*(1 + r/52)^(52*10).

(a) Find the rate of change of A wrt r.

We shall work a more general problem - assuming the interest is compunded N times per year so that
A = 10000*(1 + r/N)^(N*10).

```
> A:= proc(P,r,N,T)
     description `initial P, annual rate r (decimal), compound
  N/year, T years`;
     P*(1 + r/N)^(N*T);
  end proc;

> A(P,r,N,T);
> eval(A(10000,r,52,10));
```

$A := \mathbf{proc}(P, r, N, T)$

description *`initial P, annual rate r (decimal), compound N/year, T years`*;

$P*(1+r/N)\hat{}(N*T)$

end proc

$$P\left(1+\frac{r}{N}\right)^{N\,T}$$

$$10000\left(1+\frac{1}{52}\,r\right)^{520} \tag{1}$$

```
> dA_dr:= proc(P,r,N,T)
     description `derivative of A wrt r`;
     diff(A(P,r,N,T),r);
  end proc;
> dA_dr(P,r,N,T);

> eval(dA_dr(10000,r,52,10));
```

$dA\_dr := \mathbf{proc}(P, r, N, T)$ **description** *`derivative of A wrt r`*; $diff(A(P, r, N, T), r)$ **end proc**

$$\frac{P\left(1+\frac{r}{N}\right)^{N\,T}\,T}{1+\frac{r}{N}}$$

$$100000\left(1+\frac{1}{52}\,r\right)^{519} \tag{2}$$

(b) Find the percentage rate of change of A wrt r when r = 0.05

```
> Percent_Rate:= proc(P,r,N,T)
     description `percentage rate of change A wrt r`;
     100*( dA_dr(P,r,N,T))/(A(P,r,N,T));
  end proc;
```

Percent_Rate := **proc**(*P*, *r*, *N*, *T*) (3)

description *`percentage rate of change A wrt r`*;

100 * *dA_dr*(*P*, *r*, *N*, *T*) / *A*(*P*, *r*, *N*, *T*)

end proc

```
> Percent_Rate(P,r,N,T);
```

Notice that the answer does not depend on P; (r/N) is usually very small.

```
> Percent_Rate(10000,r,52,10);
  eval(Percent_Rate(10000,r,52,10),r = 0.05);
```

$$\frac{100\,T}{1+\frac{r}{N}}$$

$$\frac{1000}{1+\frac{1}{52}\,r}$$

$$999.0393857 \tag{4}$$

```
> limit(Percent_Rate(1,r,N,T),N=infinity);
```

$$100\,T \tag{5}$$