math16504.mw
Maple 9.5 Worksheet for Problems in Math 165 - Calculus for Business.

First load plots and student:

```
> restart:with( student):with (plots):
```

Chapter 2 Section 2.1 Problem 37. p. 111 PROFIT

A manufacturer can produce tape recorders at a cost of 20 dollars apiece. It is estimated that
if the tape recorders are sold for p dollars apiece, consumers will buy q = 120 - p recorders each month.

(a) Express the manufacturer's profit as a function of q.

(b) What is the average rate of profit obtained as the level of production increases from
q = 0 to q = 20?

(c) At what rate is the profit changing when q = 20 recorders are produced?
Is the profit increasing or decreasing at this level of production?

To answer (a), Profit(q) = Revenue(q) - Cost(q),

  Revenue(q) = (quantity) x (price) = q p = q *(120 - q),

  Cost(q) = 20*(q),

 Since Cost/item is constant = 20, an alternate expression is Cost(q) = (p - 20) q = 100 -q.

```
> Revenue:= proc(q) description`Revenue at price q`;
>     q*(120 - q):
> end proc; Revenue(q);
```

$$Revenue := \textbf{proc}(q)\ \textbf{description}\ Revenue\ at\ price\ q;\ q*(120-q)\ \textbf{end proc}$$

$$q\ (120-q)$$

```
> Cost:= proc(q) description`Cost at price q`;
>     20 * q:
> end proc;Cost(q);
>
```

$$Cost := \textbf{proc}(q)\ \textbf{description}\ Cost\ at\ price\ q;\ 20*q\ \textbf{end proc}$$

$$20\ q$$

```
> Profit:= proc(q) description`Profit at price q`;
>     Revenue(q) - Cost(q):
> end proc;Profit(q);
```
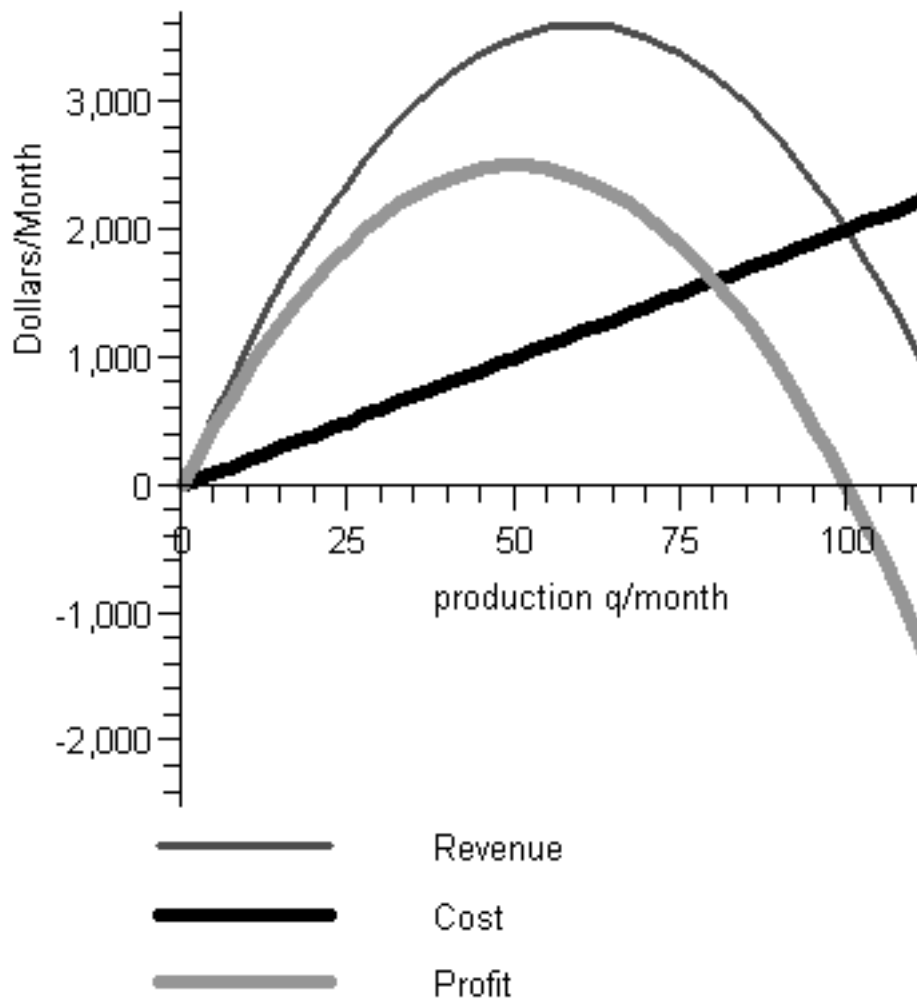
$$Profit := \textbf{proc}(q)\ \textbf{description}\ Profit\ at\ price\ q;\ Revenue(q)-Cost(q)\ \textbf{end proc}$$

$$q\ (120-q)-20\ q$$

```
> simplify([Revenue(q),Cost(q),Profit(q)]);
```

$$\left[-q\ (-120+q),\ 20\ q,\ 100\ q-q^2 \right]$$

```
> plot([Revenue(q),Cost(q),Profit(q)],q = 0..120, color=[red,black,
  green], thickness = [2,3,4],legend=[Revenue,Cost,Profit],labels =
  [`production q/month`, `Dollars/Month`],labeldirections=
  [horizontal,vertical]);
```


(b) What is the average rate of profit obtained as the level of production increases from q = 0 to q = 20?

Here there is ambiguity: "average rate of profit" (wrt time)?" is (Profit(20)+Profit(0))/20. "Average rate of change of total [monthly profit wrt q" is (Profit(20) - Profit(0))/20. "monthly profit" would be more precise!

The two answers are the same since we have a line!

To answer (b), calculate the average (Profit(20) - Profit(0))/20 :
```
> Profit(20); Profit(0);(Profit(20)+Profit(0))/10;
  Average:= (Profit(20) - Profit(0))/20;
```
$$1600$$
$$0$$
$$160$$

$$Average := 80$$

To answer (c), we calculate the derivative of the Profit function wrt the variable q at q = 20.

```
> Profitmarginalchange:= proc(q) description`Rate of Change of
  Profit at price q`;
>    diff(Profit(q),q):
> end proc; Profitmarginalchange(q);
```

$Profitmarginalchange := \mathbf{proc}(q)$

  $\mathbf{description}\ Rate\ of\ Change\ of\ Profit\ at\ price\ q;$

  $diff\,(Profit(q), q)$

end proc

$$100 - 2\,q$$

```
> eval(Profitmarginalchange(q),q = 20);
```
When q = 20, Profit is increasing at rate of 60 dollars per production/month.

$$60$$

Of course the interesting problem is to determine the maximum profit. Let's look again at the Profit function Profit(q).

```
> Profit(q);factor(Profit(q));
```

$$q\,(120 - q) - 20\,q$$

$$-q\,(-100 + q)$$

```
> plot([Profit(q)],q = 0..120, color=[green], thickness = [4],
  legend=[Profit],labels = [`production q/month`,`Profit/month`],
  labeldirections=[horizontal,vertical]);
```
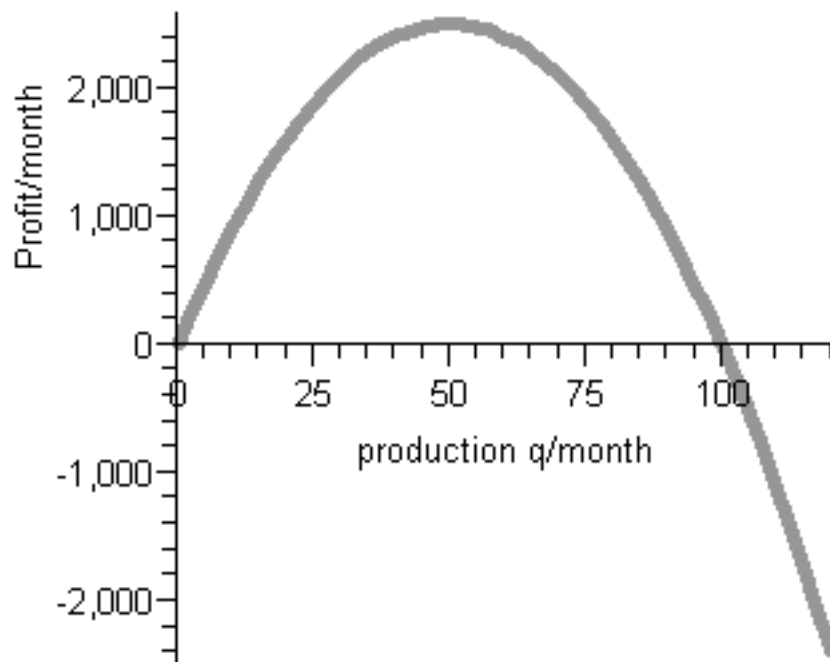
Profit

The maximum seems to occur at about q = 50, about halfway between the roots of the equation Profit (q) =0.

In general the maximum [minimum] of a function may occur only where the rate of change, or derivative is 0 --
at a maximum , a function cannot have a positive derivative - we could get a bigger value by moving to the right AND
at a maximum, a function cannot have a negative derivative - we could get a bigger value by looking to the left.

So the place to look for a maximum is to solve the equation Profit'(q) = 0.

```
> solve( Profitmarginalchange(q) = 0,q);
```
                                50