165Section3.2.mw
Maple 10 Worksheet Math 165 - Calculus for Business.
Graphs for Problems in Section 3.2
Maple does al the work on the HARD PART -- solving dy/dx = 0, etc.
Problems 3.2.7, 3.2.9, 3.2.13, 3.2.25
Example 3.2.3.
To understand the graph, you need to find a reasonable viewing window. I start with
Zstd, the standard viewing window, and then adjust.

First load plots and student

```
> restart:with(plots):with(student):with(plots):with(plottools):with
  ( RealDomain ):
```

N.B. A Maple command such as eval(f(x),x=2) is the instruction
``Evaluate f(2)'' or
``evaluate the function f(x) at x = 2.''
a:= b assigns the value 'b' to the name 'a'
Use a semicolon ; to execute a 'Maple Command' and display the output.
Using a colon : executes a command without displaying the output. (useful when you generate instructions
for plots)
I (JL) usually write functions as a procedure such as the "x square funtion":
square_function:= proc(x);x^2 ; end proc;
Other ways to write functions:
square_function:= x -> x^2;
Writing
square_function(x):= x^2;
may not work when operations such as differentiation and integration are applied.
`%` is the last computed expression (Simlar to ANS on your calculator).

Set the standard viewing window

```
> Zstd:= x=-10..10,y= -10..10;#plot(x, Zstd);
```

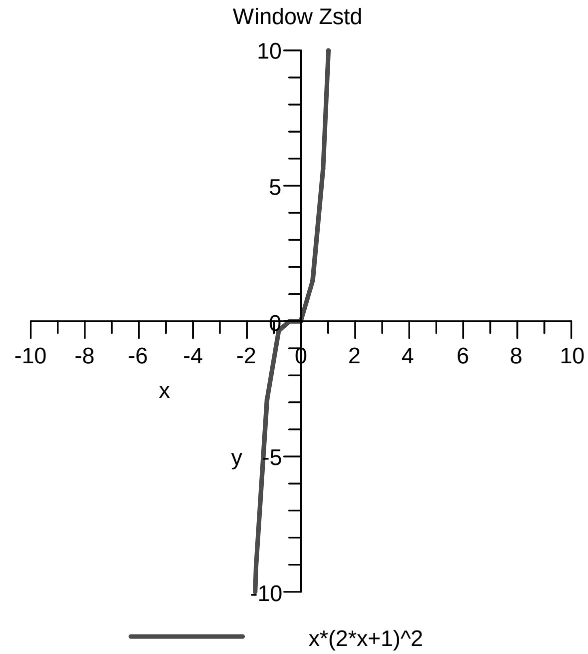
$$Zstd := x = -10..10, y = -10..10 \qquad (1)$$

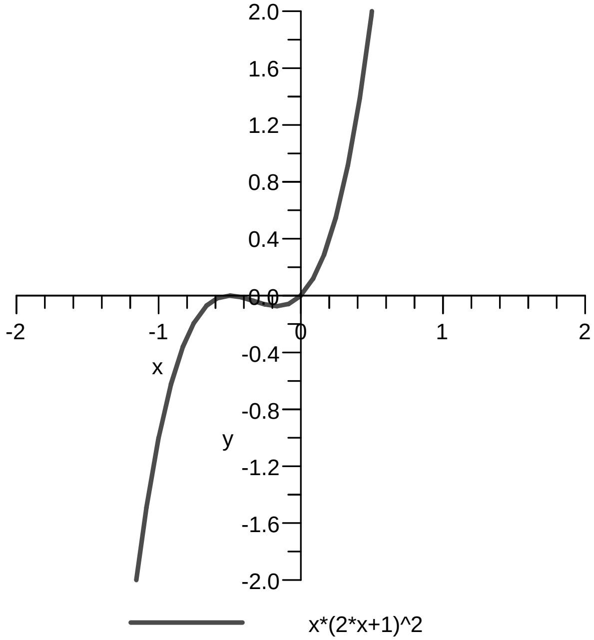
Prob3.2.7

```
f_7 := proc (x); x*(2*x+1)^2 end proc:
`f_7(x) `:=f_7(x);

> plot_7:=plot(f_7(x),Zstd,thickness=2,legend =convert(f_7(x),
  string),title=`Window Zstd` ):display(plot_7);
  plot_7a:=
  plot(f_7(x),x=-2 .. 2,y=-2 .. 2, thickness=2,legend=convert(f_7(x)
  ,string)):
  display(plot_7a);
```

$$f\_7(x) := x\,(2\,x+1)^2$$






```
> `f_7(t) `:=f_7(t);
  deriv:=proc(t);diff(f_7(t),t)  ; end proc:`deriv `:= deriv(t);
> `OR factored `:= factor(%);
  deriv_2:= proc(t); diff(deriv(t),t)  ; end proc:
  `deriv_2 `:=deriv_2(t);
  CRIT_NOS:=solve(deriv(t)= 0,t);#There may be multiple CRIT_NOS
  #test := f_7(-1/6);
  CRIT_POINTS_ON_GRAPH:=map( x->[x,f_7(x)], {CRIT_NOS} );
  #apply the fn/proc to all x in the SET of CRIT_NOS
  POSSIBLE_INFLECTION:=solve(deriv_2(t)=0,t);
  POSSIBLE_INFL_ON_GRAPH:=map( x->[x,f_7(x)], {POSSIBLE_INFLECTION}
  );
  INC:=solve(deriv(t) > 0,t);
  DEC:=solve(deriv(t) <0,t);
  CONCAVE_UP:=solve(deriv_2(t)>0,t);
  CONCAVE_DOWN:=solve(deriv_2(t)<0,t);
  plot(f_7(x),x=-2 .. 2,y=-2 .. 2, thickness=2,legend=convert(f_7(x)
  ,string)):
```

```
display(plot_7a);
```

$$f\_7(t) := t\,(2\,t+1)^2$$

$$deriv := (2\,t+1)^2 + 4\,t\,(2\,t+1)$$

$$OR\ factored := (2\,t+1)\,(6\,t+1)$$

$$deriv\_2 := 24\,t+8$$

$$CRIT\_NOS := -\frac{1}{2}, -\frac{1}{6}$$

$$CRIT\_POINTS\_ON\_GRAPH := \left\{\left[-\frac{1}{2}, 0\right], \left[-\frac{1}{6}, -\frac{2}{27}\right]\right\}$$

$$POSSIBLE\_INFLECTION := -\frac{1}{3}$$

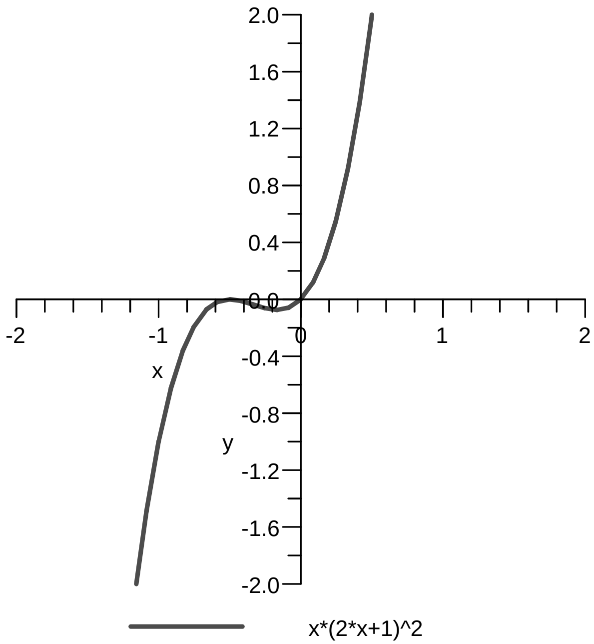
$$POSSIBLE\_INFL\_ON\_GRAPH := \left\{\left[-\frac{1}{3}, -\frac{1}{27}\right]\right\}$$

$$INC := RealRange\left(-\infty, Open\left(-\frac{1}{2}\right)\right), RealRange\left(Open\left(-\frac{1}{6}\right), \infty\right)$$

$$DEC := RealRange\left(Open\left(-\frac{1}{2}\right), Open\left(-\frac{1}{6}\right)\right)$$

$$CONCAVE\_UP := RealRange\left(Open\left(-\frac{1}{3}\right), \infty\right)$$

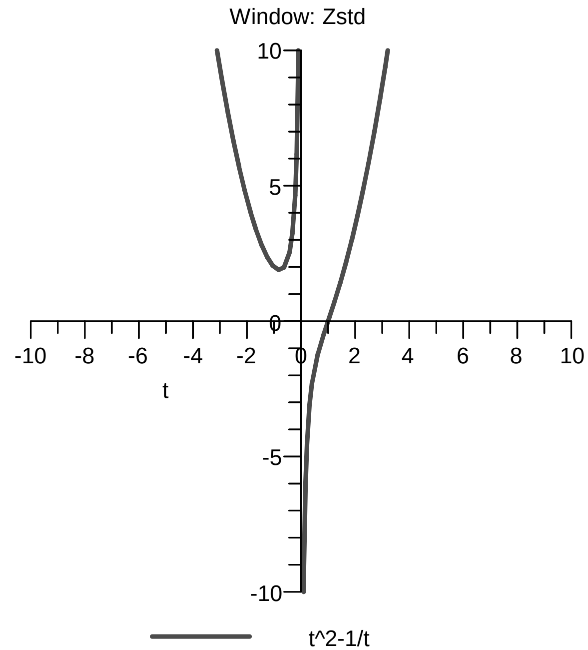
$$CONCAVE\_DOWN := RealRange\left(-\infty, Open\left(-\frac{1}{3}\right)\right)$$

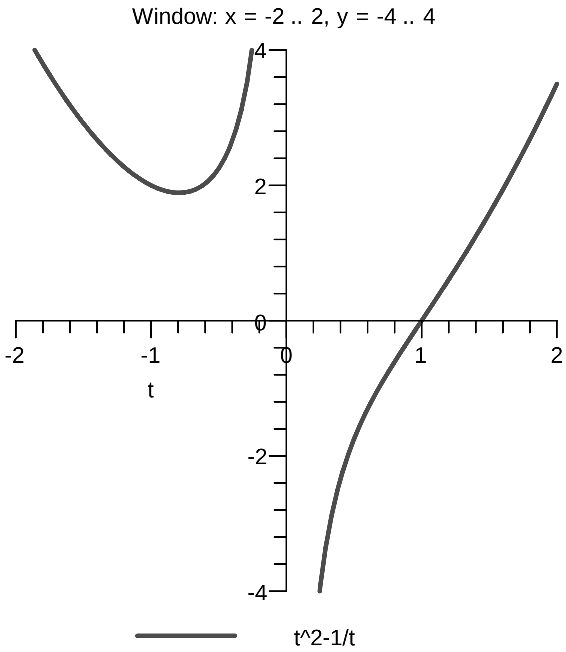


Prob 3.2.9

```
> g_9:= proc(t);
      t^2 - 1/t;
  end proc:
  `g_9(t) `:=g_9(t);
  plot(g_9(t), t= -10..10,-10..10,thickness=2,discont=true, legend=
  convert(g_9(t),string),
  title= `Window: Zstd` );
  plot(g_9(t), t= -2..2,-4..4,thickness=2,discont=true,legend=
  convert(g_9(t),string),
  title = `Window: x = -2 .. 2, y = -4 .. 4`);
```

$$g\_9(t) := t^2 - \frac{1}{t}$$






```
> `g_9(t) `:=g_9(t);
  deriv:=proc(t);diff(g_9(t),t)  ; end proc:`deriv `:= deriv(t);
> deriv_2:= proc(t); diff(deriv(t),t)  ; end proc:
  `deriv_2 `:=deriv_2(t);
  CRIT_NOS:=solve(deriv(t)= 0,t);
  CRIT_POINTS_ON_GRAPH:=map( x->[x,g_9(x)], {CRIT_NOS} );
  POSSIBLE_INFLECTION:=solve(deriv_2(t)=0,t);
  POSSIBLE_INFL_ON_GRAPH:=map( x->[x,g_9(x)], {POSSIBLE_INFLECTION}
  );
  INC:=solve(deriv(t) > 0,t);
  DEC:=solve(deriv(t) <0,t);
  CONCAVE_UP:=solve(deriv_2(t)>0,t);
  CONCAVE_DOWN:=solve(deriv_2(t)<0,t);
  plot(g_9(t), t= -2 .. 2,-4 .. 4,thickness=2,discont=true,legend=
  convert(g_9(t),string),
  title = `Window: x = -2 .. 2, y = -4 .. 4`);
```

$$g\_9(t) := t^2 - \frac{1}{t}$$

$$deriv := 2\,t + \frac{1}{t^2}$$

$$deriv\_2 := 2 - \frac{2}{t^3}$$

$$CRIT\_NOS := -\frac{1}{2}\,2^{2/3}$$

$$CRIT\_POINTS\_ON\_GRAPH := \left\{\left[-\frac{1}{2}\,2^{2/3},\ \frac{3}{2}\,2^{1/3}\right]\right\}$$

$$POSSIBLE\_INFLECTION := 1$$

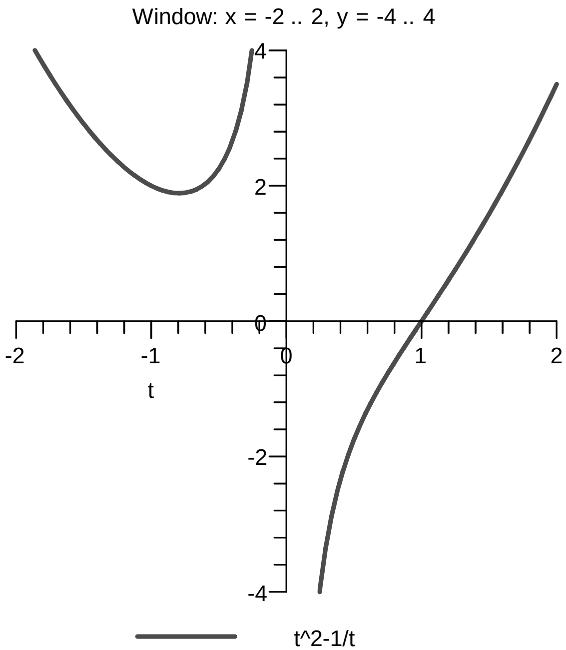
$$POSSIBLE\_INFL\_ON\_GRAPH := \{[1, 0]\}$$

$$INC := RealRange\left(Open\left(-\frac{1}{2}\,2^{2/3}\right), Open(0)\right), RealRange(Open(0),\ \infty)$$

$$DEC := RealRange\left(-\infty, Open\left(-\frac{1}{2}\,2^{2/3}\right)\right)$$

$$CONCAVE\_UP := RealRange(-\infty, Open(0)), RealRange(Open(1),\ \infty)$$

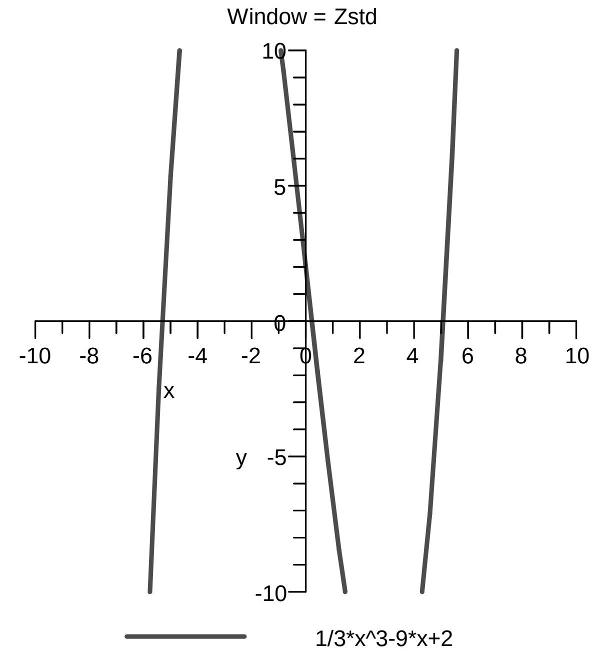
$$CONCAVE\_DOWN := RealRange(Open(0), Open(1))$$

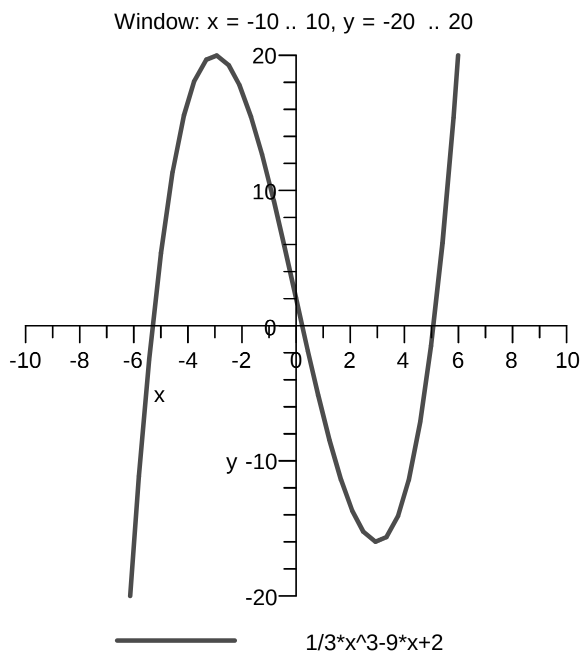


Prob 3.2.13

```
> f_13:=proc(x);
      (1/3)*x^3 - 9*x +2;
  end proc:
  `f_13(x) `:=f_13(x);
  plot(f_13(x),Zstd,thickness=2,legend=convert(f_13(x),string),
     title= `Window = Zstd`);
  plot(f_13(x),x=-10 .. 10, y = -20  .. 20,thickness=2,legend=
  convert(f_13(x),string),
  title = `Window: x = -10 .. 10, y = -20  .. 20`);
```

$$f\_13(x) := \frac{1}{3}x^3 - 9x + 2$$






```
> `f_13(t) `:=f_13(t);
  deriv:=proc(t);diff(f_13(t),t)  ; end proc:`deriv `:= deriv(t);
> deriv_2:= proc(t); diff(deriv(t),t)  ; end proc:
  `deriv_2 `:=deriv_2(t);
  CRIT_NOS:=solve(deriv(t)= 0,t);
  CRIT_POINTS_ON_GRAPH:=map( x->[x,f_13(x)], {CRIT_NOS} );
  POSSIBLE_INFLECTION:=solve(deriv_2(t)=0,t);
  POSSIBLE_INFL_ON_GRAPH:=map( x->[x,f_13(x)], {POSSIBLE_INFLECTION}
  );
  INC:=solve(deriv(t) > 0,t);
  DEC:=solve(deriv(t) <0,t);
  CONCAVE_UP:=solve(deriv_2(t)>0,t);
  CONCAVE_DOWN:=solve(deriv_2(t)<0,t);
  plot(f_13(x),x=-10 .. 10, y = -20  .. 20,thickness=2,legend=
  convert(f_13(t),string),
  title = `Window: x= -10 .. 10, y = -20  .. 20`);
```

$$f\_13(t) := \frac{1}{3}\, t^3 - 9\, t + 2$$

$$deriv := t^2 - 9$$

$$deriv\_2 := 2\, t$$

$$CRIT\_NOS := 3, -3$$

$$CRIT\_POINTS\_ON\_GRAPH := \{[3, -16], [-3, 20]\}$$

$$POSSIBLE\_INFLECTION := 0$$

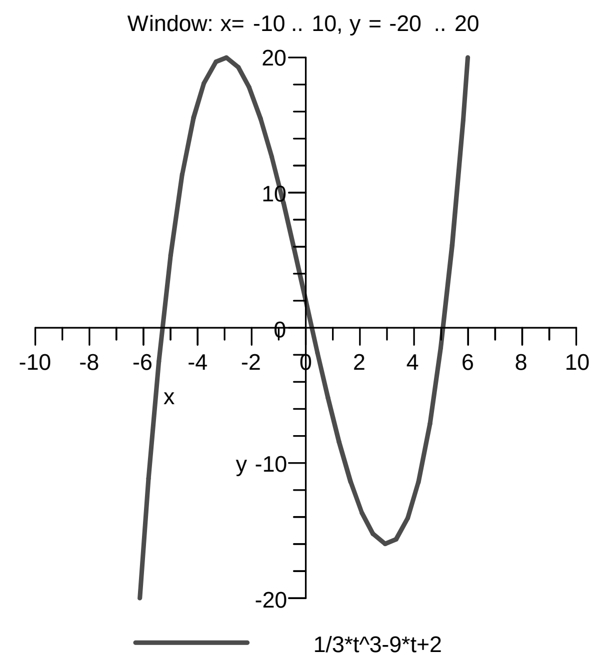
$$POSSIBLE\_INFL\_ON\_GRAPH := \{[0, 2]\}$$

$$INC := RealRange(-\infty, Open(-3)), RealRange(Open(3), \infty)$$

$$DEC := RealRange(Open(-3), Open(3))$$

$$CONCAVE\_UP := RealRange(Open(0), \infty)$$

$$CONCAVE\_DOWN := RealRange(-\infty, Open(0))$$

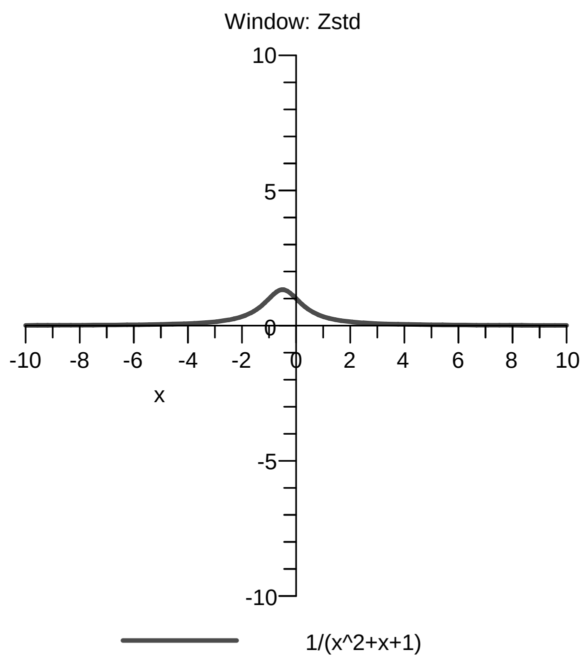


Prob 3.2.25

```
> f_25:= proc(x);
     1/(x^2 + x + 1);
  end proc:
```

```
`f_25(x) `:= f_25(x);
plot(f_25(x), x= -10 .. 10, -10..10, thickness=2,legend=convert
(f_25(x),string),
title = `Window: Zstd`);
plot(f_25(x), x= -4..4, -2..2, thickness=2,legend=convert(f_25(x),
string),
title = `Window: x= -4 .. 4, y = -2 .. 2`):
```

$$f\_25(x) := \frac{1}{x^2+x+1}$$




```
> `f_25(t) `:=f_25(t);
deriv:=proc(t);diff(f_25(t),t)  ; end proc:`deriv `:= deriv(t);
> deriv_2:= proc(t); diff(deriv(t),t)  ; end proc:
`deriv_2 `:=deriv_2(t);`normalized `:= simplify(deriv_2(t));
CRIT_NOS:=solve(deriv(t)= 0,t);
CRIT_POINTS_ON_GRAPH:=map( x->[x,f_25(x)], {CRIT_NOS} );
POSSIBLE_INFLECTION:=solve(deriv_2(t)=0,t);
POSSIBLE_INFL_ON_GRAPH:=map( x->[x,f_25(x)], {POSSIBLE_INFLECTION}
);
INC:=solve(deriv(t) > 0,t);
DEC:=solve(deriv(t) <0,t);
CONCAVE_UP:=solve(deriv_2(t)>0,t);
CONCAVE_DOWN:=solve(deriv_2(t)<0,t);
plot(f_25(x), x= -4..4, -2..2, thickness=2,legend=convert(f_25(x),
string),
title = `Window: x = -4 .. 4, y = -2 .. 2`);
```

$$f\_25(t) := \frac{1}{t^2+t+1}$$

$$deriv := -\frac{2\,t+1}{\left(t^2+t+1\right)^2}$$

$$deriv\_2 := \frac{2\,(2\,t+1)^2}{\left(t^2+t+1\right)^3} - \frac{2}{\left(t^2+t+1\right)^2}$$

$$normalized := \frac{6\,t\,(t+1)}{\left(t^2+t+1\right)^3}$$

$$CRIT\_NOS := -\frac{1}{2}$$

$$CRIT\_POINTS\_ON\_GRAPH := \left\{\left[-\frac{1}{2}, \frac{4}{3}\right]\right\}$$

$$POSSIBLE\_INFLECTION := 0, -1$$

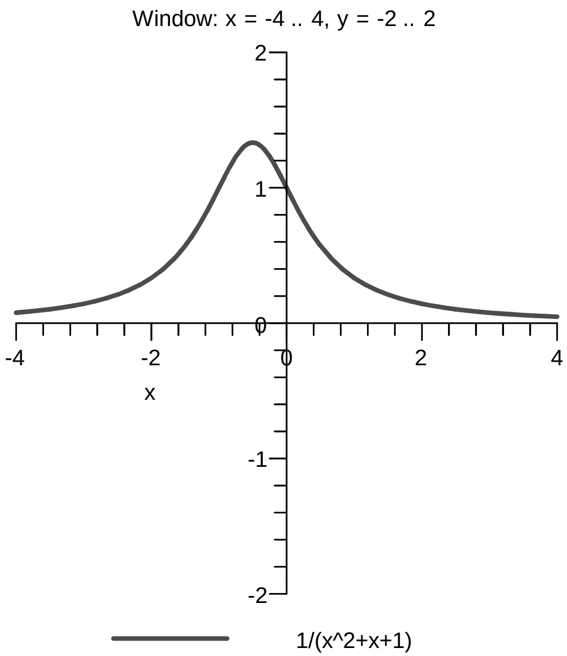
$$POSSIBLE\_INFL\_ON\_GRAPH := \{[0, 1], [-1, 1]\}$$

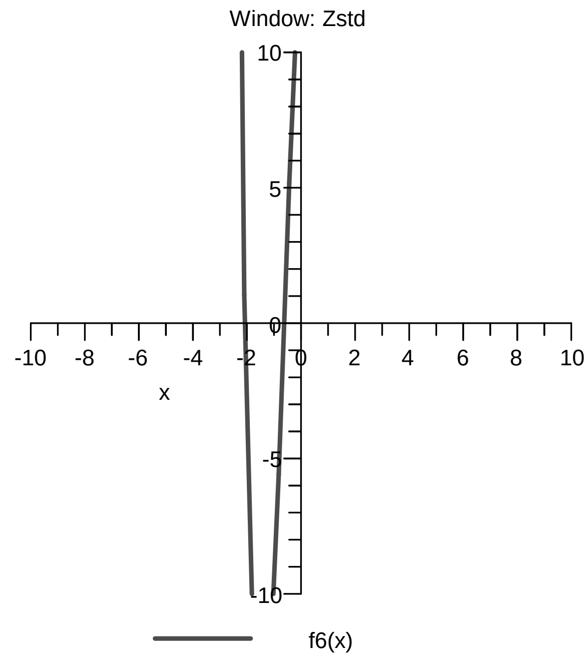
$$INC := RealRange\left(-\infty, Open\left(-\frac{1}{2}\right)\right)$$

$$DEC := RealRange\left(Open\left(-\frac{1}{2}\right), \infty\right)$$

$$CONCAVE\_UP := RealRange(-\infty, Open(-1)), RealRange(Open(0), \infty)$$

$$CONCAVE\_DOWN := RealRange(Open(-1), Open(0))$$

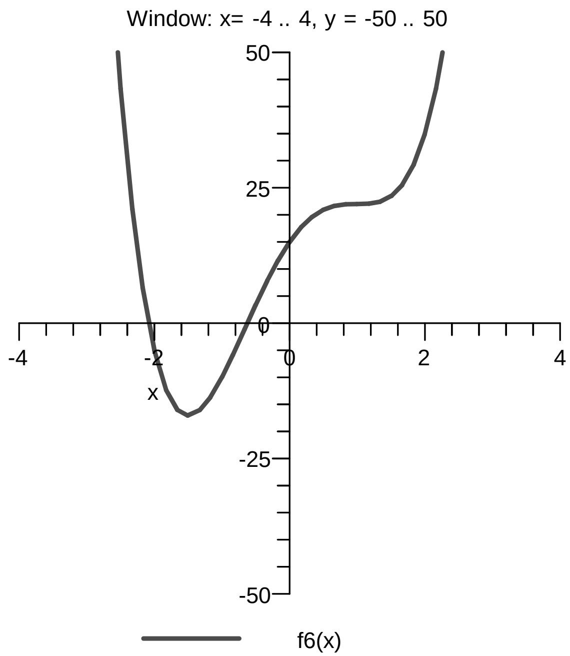




Example 3.2.3

```
> f:= proc(x);
     3*x^4 - 2* x^3 -12 * x^2 + 18*x +15;
  end proc:
  `f(x) `:= f(x);
  plot(f(x), x= -10 .. 10, -10..10, thickness=2,legend=convert(f6(x)
  ,string),
  title = `Window: Zstd`);
  plot(f(x), x= -4..4, -50 .. 50, thickness=2,legend=convert(f6(x),
  string),
  title = `Window: x= -4 .. 4, y = -50 .. 50`);
```

$$f(x) := 3\,x^4 - 2\,x^3 - 12\,x^2 + 18\,x + 15$$




```
> `f(t) `:=f(t);
> deriv:=proc(t);diff(f(t),t)  ; end proc:`deriv `:= deriv(t);
  `OR the hard part `:=factor(%);
> deriv_2:= proc(t); diff(deriv(t),t)  ; end proc:
  `deriv_2 `:=deriv_2(t);`normalized `:= simplify(deriv_2(t));
   `OR the hard part `:=factor(%);
  CRIT_NOS:=solve(deriv(t)= 0,t);#test:=f(-3/2);
  CRIT_POINTS_ON_GRAPH:=map( x->[x,f(x)], {CRIT_NOS} );
  POSSIBLE_INFLECTION:=solve(deriv_2(t)=0,t);
  POSSIBLE_INFL_ON_GRAPH:=map( x->[x,f(x)], {POSSIBLE_INFLECTION} );
  INC:=solve(deriv(t) > 0,t);
  DEC:=solve(deriv(t) <0,t);
  CONCAVE_UP:=solve(deriv_2(t)>0,t);
  CONCAVE_DOWN:=solve(deriv_2(t)<0,t);
  plot(f(x), x= -4..4, -50 .. 50, thickness=2,legend=convert(f(x),
  string),
  title = `Window: x= -4 .. 4, y = -50 .. 50`);
```

$$f(t) := 3\,t^4 - 2\,t^3 - 12\,t^2 + 18\,t + 15$$

$$deriv := 12\,t^3 - 6\,t^2 - 24\,t + 18$$

$$OR\ the\ hard\ part := 6\,(2\,t + 3)\,(t - 1)^2$$

$$deriv\_2 := 36\,t^2 - 12\,t - 24$$

$$normalized := 36\,t^2 - 12\,t - 24$$

$$OR\ the\ hard\ part := 12\,(3\,t + 2)\,(t - 1)$$

$$CRIT\_NOS := -\frac{3}{2},\ 1,\ 1$$

$$CRIT\_POINTS\_ON\_GRAPH := \left\{[1, 22], \left[-\frac{3}{2}, -\frac{273}{16}\right]\right\}$$

$$POSSIBLE\_INFLECTION := 1, -\frac{2}{3}$$

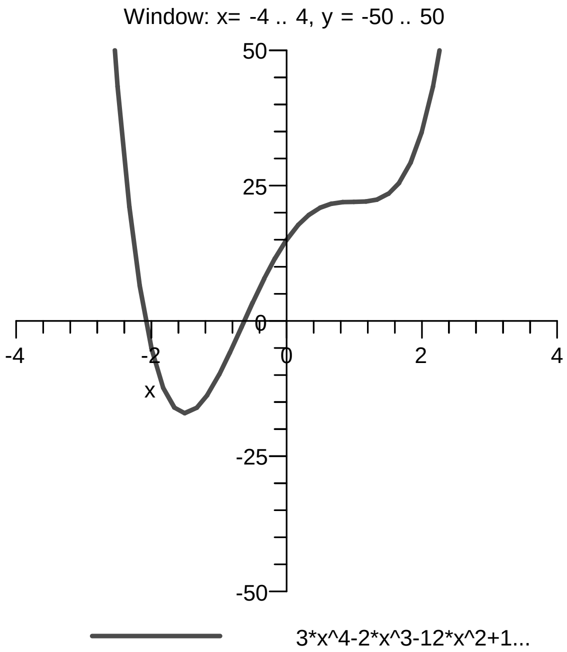
$$POSSIBLE\_INFL\_ON\_GRAPH := \left\{[1, 22], \left[-\frac{2}{3}, -\frac{31}{27}\right]\right\}$$

$$INC := RealRange\left(Open\left(-\frac{3}{2}\right), Open(1)\right), RealRange(Open(1),\ \infty)$$

$$DEC := RealRange\left(-\infty, Open\left(-\frac{3}{2}\right)\right)$$

$$CONCAVE\_UP := RealRange\left(-\infty, Open\left(-\frac{2}{3}\right)\right), RealRange(Open(1),\ \infty)$$

$$CONCAVE\_DOWN := RealRange\left(Open\left(-\frac{2}{3}\right), Open(1)\right)$$


Window: x= -4 .. 4, y = -50 .. 50
50
25
0
-25
-50
-4
-2
0
2
4
x
3*x^4-2*x^3-12*x^2+1...