p.291 Power function of x $\quad \underbrace{x}_{\text{variable}}^{3/2} \leftarrow$ fixed

Exponential fn of x $\quad a^x \leftarrow$ power/exponent changes; $a$ fixed

Rules for exponential functions $y = a^x$,

$a \neq 0$ ($a = 1$) interpret as $y = 1$ (not

sum of exponents "go to products" $\quad a^{x+y} = (a^x)(a^y)$

repeated $\quad (a^x)^r = a^{(x \cdot r)}$

Negative $\quad a^{-x} = \frac{1}{a^x} \quad \left[ a^0 = 1 = a^{x+(-x)} = a^x (a^{-x}) ; \; a^{-x} = \frac{1}{a^x} \right.$

differences $\quad a^{r-s} = \frac{a^r}{a^s}$

Does one consider $a$ negative:

$y = a^x$ is the exponential fn of x with base a

Special "a": $\quad a = e \equiv \lim_{N \to \infty} \left(1 + \frac{1}{N}\right)^N \approx 2.718281828$ (not repeating!

(Universal $e^x$ means this particular) for calculatation $\exp(x)$; calculators, typing, etc

Exponential Growth & Decay Functions

Model: Rate of change of stuff is proportional to amount of stuff.

$$f(x+h) - f(x) = C_h f(x)$$

$C_h$ Positive ($h$) $\to$ growth. Every $h$ multiply by $C$

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(usual variable is $t$ = "time")

$f(t+h) - f(t) = C_h f(t)$ DISCRETE argum

$= \sum_{k=} f(t+k\frac{h}{N}) - f(t+\frac{k-1}{N}) = C_{\frac{h}{N}}^N f(t)$

$\lim C_{\frac{h}{N}}^N$ exists. (can be more rigorous)

(better when we look at derivatives

$$Q(t) = Q_0 e^{kt}$$ $Q_0$ stuff at $t=0$ $k$

$Q$ is : amount of stuff

$k$ positive growing

$= Q_0 e^{-kt}$ ($k$ assumed

decay

Example 4-1-6 August ] do later after derivative

concentration $C = 3.97\, e^{-.9t}$

$\cancel{C(2) - C}$ $\frac{C(t+1) - C(t)}{C(t)} = \frac{3.97}{3.97} \frac{[e^{-.9(t+1)} - e^{-.9t}]}{e^{-.9t}}$

$= 3.97 \left[e^{-.9} - 1\right]$

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Exponential Growth of Bacteria

$\cancel{e}Q = \cancel{e^{kt}} Q_0 e^{kt}$ $\quad Q(t+h) = Q_0 e^{k(t+h)}$

$= \underbrace{e^{kh}}_{\text{constant [depending on h}} (Q_0(t))$

$\boxed{\text{Prob 43}}$

| $Q$ $t$ minutes | 0 | 10 |
|---|---|---|
| $Q$ no. cells | 5000 | 8000 |

Function $Q = \underbrace{8000}_{\text{ini}} \left(\frac{8000}{5000}\right)^t$ multiples by constant $\frac{8000}{5000}$

$Q(t) = \underbrace{5000}\ \underbrace{a}^{t}$ $(= 5000\ e^{kt})$

$\frac{8200}{\cancel{8000}}$ [N.B. >1]

$e^{kt} = a^t = (e^k)^t$

$\boxed{a = e^k}$ NEED to have a [fixed way] to solve this equation

~~E~~ Another example (Sec. 4.2)

Want 1000 to grow to 2000. How long at 5%

$FV(t) = 1000\, e^{.05t}$

Solve $= 2000 = 2(1000)$

Solve for $t$ $e^{.05t} = 2$

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logarithm functions

General Defn "$\log_b$": "log base b" "log-sub-b"

~~$b \neq 0$~~, $b > 0$, $b \neq 1$

$\log_b(x)$ is the solution of $b^{?} = x$

Most important: $b=10$ (computation with real numbers)

$b=e$ $\log_e(x) \rightarrow \ln(x)$ ← special.

$\log(x)$ ← scientific

log usually means $\begin{cases} \ln \text{ or } \log_e \text{ in scientific} \\ \log_{10} \text{ in computation/commercial} \end{cases}$

(they are related - Fortunately ln is "computable" (without resort to 2.71828...)

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Properties

Products good: $\ln(rs) = \ln(r) + \ln(s)$

Careful: you are assuming both r and s >0

~~Sums (not simple~~ ln (sums) nothing simple

Exponents (Powers) $\ln(x^r) = r \cdot \ln(x)$

Combinations

$$\ln(x^{12} y^{-1/2})$$
$$= 12 \cdot \ln(x) - \tfrac{1}{2}\ln(y)$$