Cf Notes on max/min, avg cost

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Extrema (MAX / MIN) of y cont on.

closed interval < CR NOS / F
END

open interval — < CR NOS
MAY NOT EXIST

$y = x + \frac{1}{x}$ 1° $x > 0$ CR NOS. $x = +1$ (also -1 but not in domain)
NO MAX

2° $[\frac{1}{2}, 2]$ CR NO. ENDS

ABMIN at 1 (can see)
ABS MAX at [both END POINTS

3° $[2,4]$ CR NOS outside END

4° $(2,4)$ NO MAX NO MIN

5° $[1,4]$

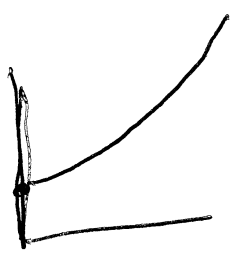
Problems 1. Max Revenue for $q = 240 - 2b$

$R = b \cdot q = b(240 - 2b)$ find b

$0 \le b \le 120$ of interest

2. Average Cost

$C(x)$ cost of producing first x units



$A(x) = \frac{C(x)}{x}$: Average Price of first x units

Minimize $A(x)$, $0 < x < +\infty$

$\frac{dA}{dx} = \frac{d}{dx}\frac{C(x)}{x} = \frac{xC'(x) - 1C(x)}{x^2} = 0$ (for $x > 0$)

when $xC'(x) = C(x)$ or $C'(x) = \frac{A(x)}{x}$

Problems 3.4:

5. ABS MAX min on

$\#5 = 3t^5 - 5t^3, \quad -2 \le t \le 0$

10

$\frac{dy}{dt} = 15t^4 - 15t^2 = 15t^2(t^2-1)$

Roots 0, 0, 1, -1

$y_1(-2) = -56 \leftarrow$ ABS MIN

$y_1(-1) = 2 \leftarrow$ ABS MAX

$y_1(0) = 0 \leftarrow$ endpoint ¿ REL MIN ?

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pp. 17 ff (a) Profit $P(q)$

Marginal Revenue $\frac{dR}{dq}$

Marginal Cost $\frac{dC}{dq}$

Maximize $P(q)$ $\left[\frac{dP}{dq}\right] = 0$

17. $p(q) = 49 - q$; $C(q) = \frac{1}{8}q^2 + 4q + 200$

Profit $q(49-q) - \left(\frac{1}{8}q^2 + 4q + 500\right)$

$\therefore \frac{dR}{dq} = 49 - 2q$;

$\therefore \frac{dC}{dq} = \frac{1}{4}q + 4$

$\frac{dP}{dq} = 0$ w.l. $\frac{dR}{dq} = \frac{dC}{dq}$

$49 - 2q = \frac{1}{4}q + 4$

$45 = \frac{9}{4}q$;

$q = \frac{180}{9} = 20.$

Average Cost $C(q) = \frac{1}{8}q^2 + 4q + 200$

$$A(q) = \frac{1}{8}q + 4 + \frac{200}{q}$$

$$\frac{dA}{dq} = \frac{1}{8} - \frac{200}{q^2}, \quad \cdots, \quad q^2 = 1600, \quad \boxed{q = 40} \text{ is MIN.}$$

$$C'(q) = \frac{1}{4}q + 4$$

$$C'(40) = \frac{1}{4} \cdot 40 + 4 = 14$$

$$A(40) = \frac{1}{8}(40) + 4 + \frac{200}{40} = 5 + 4 + 5 = 14!$$

Elasticity Definition "Price elasticity of Demand"

Ratio of Relative(small) change

$$\frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} = \frac{p}{q}\frac{\Delta q}{\Delta p} \to \boxed{\frac{p}{q}\frac{dq}{dp}}$$

$E(p)$: change p by Δp

vs. Revenue ~~R changes price~~

price changed by $+\varepsilon\%$

q changes by $\mp |E(p)|\varepsilon\%$

$$\text{New } R = R(1+\varepsilon\%)(1 \mp |E(p)|\varepsilon\%)$$

$$= R\left(1 - \underbrace{|E|\varepsilon^2}_{\text{much smaller than } \varepsilon \text{ (small!)}} + \varepsilon(1 \mp E(p))\right)$$

$|E(b)| > 1 \quad (E(b) < -1)$

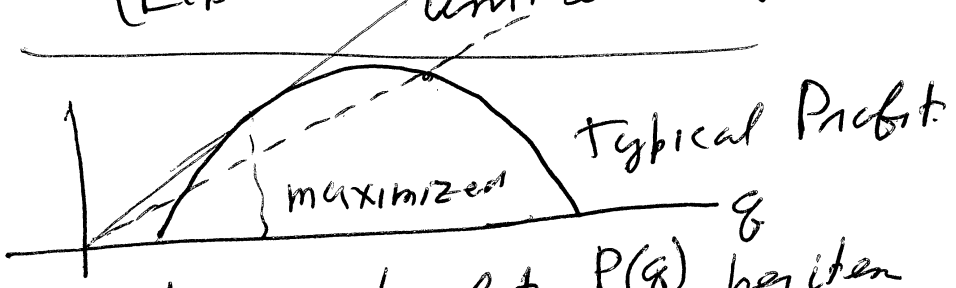
Elastic Demand – size of change in q larger than % change in p

R(p) decreasing

$|E(b)| < 1 \quad (1 > E(b) > -1)$

Inelastic

$|E(b)| = 1$ unitary elasticity
unit elasticity



Average profit $\frac{P(q)}{q}$ per item

$\frac{dP}{dq} = P'(q)$ (definitely not 0