24:43* 70(a)(b)

Ex 2.4.8: $f(x)=(3x+1)^4(2x-1)^5$ find x s.t. tan line is horiz

i.e. $\frac{dy}{dx}=0$

- Der: $4(3x+1)^3\cdot 3(2x-1)^5+(3x+1)^4 5\cdot(2x-1)^4\cdot 2$

$$=(3x+1)^3(2x-1)^4\{12(2x-1)+10(3x+1)\}$$

PROB: solve $=0$: retain factored

$$(3x+1)^3(2x-1)^4\{\underbrace{24x-12+30x+10}_{54x-2}\}$$

Roots at $x=-\frac{1}{3}$ (triple); $x=\frac{1}{2}$ (4th order) $x=\frac{1}{27}$ (single)

Ex 2.4.9. $y=\frac{3x-2}{(x-1)^2}$

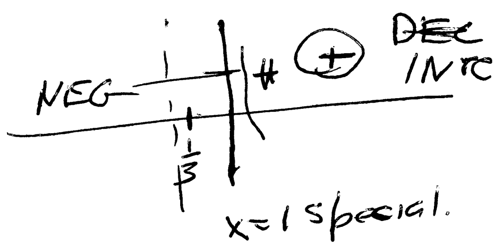
$$\frac{dy}{dx}=\frac{(x-1)^2\cdot 3-(3x-2)\cdot 2(x-1)\cdot 1}{(x-1)^4}$$

$$=\frac{(x-1)[3(x-1)-(6x-4)]}{(x-1)^4}$$

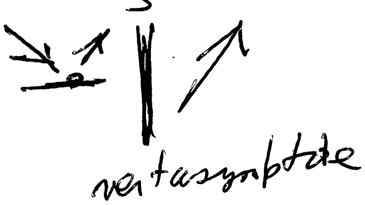
$$=\frac{3x-3-6x+4}{(x-1)^3}=\frac{1-3x}{(x-1)^3}$$

1° Where is $\frac{dy}{dx}=0$ $\boxed{x=\frac{1}{3}}$

2° Where is $\frac{dy}{dx}$ 0, ⊕ ⊖



changes sign $x=\frac{1}{3}$ HOR TAN



2.4 prob 43 tangent line horiz

$f(x) = (x^2+x)^2$

$\frac{dy}{dx} = 2(x^2+x)(2x+1)$

$= 2x(x+1)(2x+1)$

$\frac{dy}{dx} = 0$ when $x=0, x=-1, x=-\frac{1}{2}$.

(called a critical number)

2.4 prob 61 Demand $D(p)$ at price p

$D(p) = \frac{40000}{p}$; t months from now price

$p(t) = (0.4t^{3/2} + 6.8)$ \$/unit

% rate of change 4 months from now

$100 \frac{\frac{dD}{dt}}{D}$ 4 months from now

$\frac{dD}{dt} = \frac{dD}{dp} \cdot \frac{dp}{dt}$

$\frac{dD}{dp} = -\frac{40000}{p^2}$ [Quot Rule not Necessary!]

$\frac{dp}{dt} = 3 \times 0.4 t^{1/2}$

Answer $100 \frac{-\frac{40000}{p^2} \cdot \underbrace{3 \times 0.4 \cdot 4^{1/2}}_{2.4}}{\cancel{p} - \frac{40000}{p}} = -\frac{100}{p} \cdot .4 \cdot 4^{1/2} = -24\%?$

N.B. $p = \cancel{3 \times} 0.4t^{3/2} + 6.8 \Big|_{t=4} = .4 \cdot 8 + 6.8 = 10$

ANS $100 \cancel{\times -\frac{40000}{10^3}} \cdot 4.2$

70

Compound Interest: 10000 invested at r (decimal) 20090202 31

comp weekly principle after 10 years will be

$$A = \underbrace{10000}_{\text{initial}} \left(1+\frac{r}{52}\right)^{520} \text{ weeks}$$

mult by every week

effect of one week multiplier

$$\frac{dA}{dr} = 10000 \cdot 520 \left(1+\frac{r}{52}\right)^{520-1} \cdot \frac{1}{52}$$

$$= \underbrace{10000}_{\text{initial}} \cdot \underbrace{10}_{\text{time}} \left(1+\frac{r}{52}\right)^{520-1}$$

$$\frac{\frac{dA}{dr}}{A} = \frac{\cancel{10000} \cdot 10 \left(1+\frac{r}{52}\right)^{520-1}}{\cancel{10000} \left(1+\frac{r}{52}\right)^{520}}$$

$$= \frac{1}{1+\frac{r}{52}} \cdot 10 \approx 10 = 1000\%$$

i. change r by 1% (.01) gives about $\frac{1}{10}$ more. (10 × .01)

1.65 m

$$\left(\begin{aligned} A &= A_0 e^{rT} \\ \frac{dA}{dr} &= T A_0 e^{rT} \end{aligned} \right\} \text{ exponential case}$$

$$\frac{\frac{dA}{dr}}{A_0 e^{rT}} = T$$

2.4.58 $C(q) = 0.2q^2 + q + 900$

$q(t) = t^2 + 100t$ after t hrs

$\left.\frac{dC}{dt}\right|_{t=1}$ $\quad \frac{dC}{dq} = 0.4q + 1$

$\frac{dq}{dt} = 2t + 100$

$$\left.\frac{dC}{dt}\right|_{t=1} = \left.0.4q+1\right|_{\substack{t=1\\q=101}} \cdot \left.\frac{dq}{dt}(2t+100)\right|_{t=1}$$

$$= ((0.4)101 + 1)(102) \approx$$

Here ~~they~~ plug in at the end.

Economics Marginal Analysis

Derivative approximates change from q to $q+1$ units produced (Cost of $(q+1)^{st}$ unit.

$$C'(x_0) \equiv \left.\frac{dC}{dx}\right|_{x=x_0} \quad \text{marginal cost}$$

$$\approx \frac{C(x_0+1) - C(x_0)}{1}$$ provided 1 is "additional cost"

"small wrt to x_0" (relatively small

[In a certain sense $C'(x_0) \approx \frac{C(x_0+\frac{1}{2}) - C(x_0-\frac{1}{2})}{1}$

will $\approx$ even closer than above or the average cost of producing x_0'th and x_0+1^{st} unit

Marginal	Additional Cost
Cost $C'(x)$	$C(x+1) - C(x)$
Revenue $R'(x) = \cancel{R(x)}$	$(x+1)q(x+1) - x\cancel{R}q(x)$
$R(x) = xq \quad \frac{dR}{dx} = q + x\frac{dq}{dx}$	$= x(q(x+1) - q(x)) + q(x+1)$
Profit $R'(x)$	$\approx P(x+1) - P(x)$