The Algebra Symposium: Fuel Efficiency

The Problem

> It is asserted that if you drive a car at constant speed; the number of miles per gallon will first increase and then decrease (or a reasonable range say 15 mph to 90 mph). Can someone explain the physics of this? Is there any sense that this function is quadratic?

After I did the note that follows, we realized that I was showing that *gallons per mile*, gpm (sic), "will first decrease and then increase." Actually we got the idea listening to the (e–mail) discussion of the nonlinear term – showing the value of cooperative work!

Minimization of Total Consumption for Any Convex Nonlinearity

If v = velocity, M = miles, $E(v)$ = time rate of fuel consumption at speed v, then the total time T to travel M miles is M/v, and the total fuel consumed, G, to travel M miles is

$$G = \frac{M}{v} * E(v).$$

Assume that $E(v) = E_0 + E_1 v + \phi(v)$, where $\phi(v)$ is the *nonlinear part*. It is reasonable to assume that $E(v)$ is a convex (nee concave up) function of v. The following observations apply to any $\phi(v)$ which is convex.

As we all know from Math 165, $E(v)/v$, is minimized when

$$E'(v) = \frac{E(v)}{v},$$

by the quotient rule for differentiation.

Geometrically, this is seen drawing the graph of $E(v)$, and then observing that the slope of the line from 0 to $(v, E(v))$ has slope $E(v)/v$; the slope is minimized when this line is tangent to the graph.

The result in Math 165 microeconomics is that the *Average Cost* of producing the first q units, $C(q)/q$, is minimized when it is the same as the *Marginal Cost*, $C'(q)$, of producing the qth unit:

$$A(q) = \mathrm{MC}(q).$$

Units Note

The original problem was about miles–per–gallon which is a constant times $\frac{M}{G} = \frac{v}{E(v)}$. Thus **minimizing** $E(v)/v$ is the same problem as **maximizing** miles–per–gallon.