Math 165 Ln–Exp Worksheet * (Key)

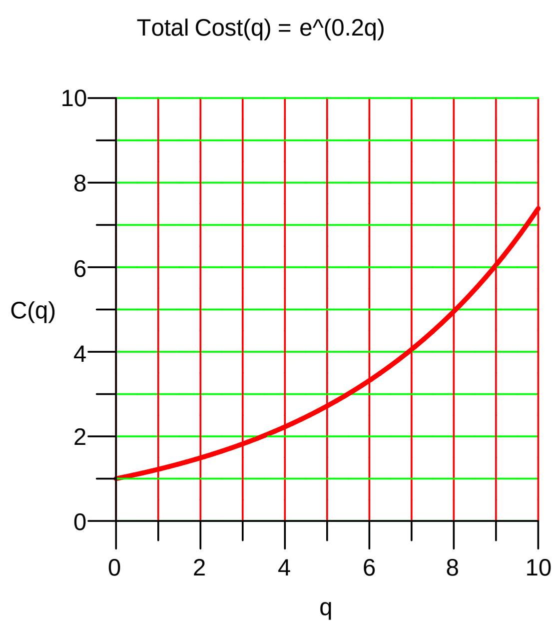
1. 9e.4.3.Pr61. The cost $C(x)$ of producing the first x units of a commodity is $C(x) = e^{0.2x}$.

a. Find the marginal cost $\dfrac{dC}{dx}$. * $0.2 * e^{0.2x}$

b. Determine the level of production x for which the marginal cost equals average cost $A(x) = \dfrac{C(x)}{x} = \dfrac{dC}{dx}$. * Solve $e^{0.2x}/x = .02e^{0.2x}$. $x = 5$. Average cost is minimized!

c. Determine the level of production x for which the average cost $A(x)$ is minimized. * $\dfrac{dA}{dx} = \dfrac{e^{0.2x}\,(.2x-1)}{x^2}$, critical number at $x = 5$.

d. Here is the graph of $C(x) = e^{0.2x}$ (I have changed x to q).



For various values of q, use a straight edge or ruler to represent lines through $O = (0, 0)$ and $(q, C(q))$. The slope of these lines is $C(q)/q = A(q)$. As you move q to the right, the slope of the line from 0 to $(q, C(q))$ decreases and then increases. Which of these lines has minimal slope? How does your answer correspond the the result in parts (b.) and (c.)?

To view an animated picture go to:

`http://www2.math.uic.edu/~lewis/165lnexp.key.htm`.

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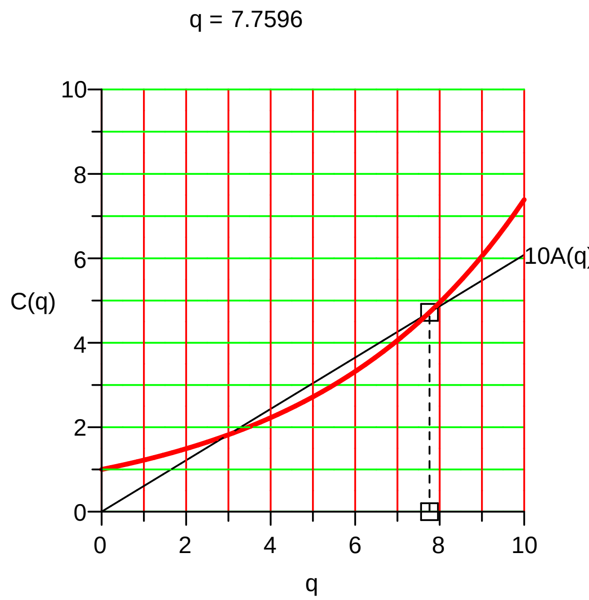

2. 9e.4.2.Pr55.Variation. **Worker Efficiency**. Worker efficiency is measured by (hourly) output Q and is given by a formula of the form

$$Q(t) = 500 - Ae^{-kt},$$

where t is the "experience" measured by the time in months after initial training.

a. What is $\lim_{t\to\infty} Q(t)$? * Since $\lim_{t\to\infty} e^{-kt} = 0$, $\lim_{t\to\infty} Q(t) = 500$.

b. It is observed that a newly trained worker has an hourly output of 300 units, and that after 6 months, a worker has an hourly output of 410 units. Find the function of the form

$$Q(t) = 500 - Ae^{-kt},$$

which fits the data.

Hint: What is the form of the function $F(t) = 500 - Q(t)$?

* $F(t) = 500 - Q(t) = Ae^{-kt}$ is an exponential decay function. Since $F(0) = 500 - 300 = 200$, and $F(6) = 90$, $F(t) = 200 * (90/200)^{t/6}$. $Q(t) = 500 - 200 * (90/200)^{t/6}$. Since $(90/200)^{1/6} = e^{\ln(90/200)^{1/6}} = e^{(1/6)\ln .45} \approx e^{-.13}$, an alternate form is $Q(t) = 500 - 200e^{-.13t}$.

3. 9e Example 4.3.14 Variation. Total net profit comes from wheat, steel, and oil.

   At a certain time,

   The profit W from wheat accounts for 20% of net profit and is increasing at a rate of 2%.

   The profit S from steel accounts for 30% of net profit and is increasing at a rate of 3%.

   The profit O from oil accounts for 50% of net profit and is decreasing at a rate of 5%.

   At what percentage rate is total net profit increasing or decreasing?

Hint: When the data is taken, find

a. $\dfrac{W}{W+S+O} =$ * .20 from the data

b. $\dfrac{\frac{dW}{dt}}{W} =$ * .02

* Similarly for S and O. You are asked to find

$$\begin{aligned}\frac{\frac{dW}{dt}+\frac{dS}{dt}+\frac{dO}{dt}}{W+S+O} &= \frac{\frac{dW}{dt}}{W}\cdot\frac{W}{W+S+O}+\cdots \\ &= (.20 * .02 + .30 * .03 - .50 * .05) \\ &= -.012 = -1.2\%\end{aligned}$$

Data and results are given as proportions.

4. 9e.4.3.Pr66. Money is deposited in a bank offering interest at an annual rate of 6% compounded continuously (CC). Find the percentage rate of change with respect to time.

* $P(t) = P(0)e^{.06t}$, $\dfrac{\frac{dP}{dt}}{P} = \dfrac{.06P(0)e^{.06t}}{P(0)e^{.06t}} = .06 = 6\%$. Notice that $P(0)e^{.06t}$ **cancels**. The percentage rate (proportion) does not depend on the initial investment.

5. 9e.4.3.Pr33. If $g(u) = \ln\left(u + \sqrt{u^2+1}\right)$, find $g'(u)$. Simplify your answer.

* $g'(u) = \dfrac{1}{u+\sqrt{u^2+1}}\cdot\left(1+\dfrac{2u}{2\sqrt{u^2+1}}\right) = \dfrac{1}{u+\sqrt{u^2+1}}\cdot\dfrac{\sqrt{u^2+1}+u}{\sqrt{u^2+1}} = \dfrac{1}{\sqrt{u^2+1}}$