Math 165: Optimizing Average Profit

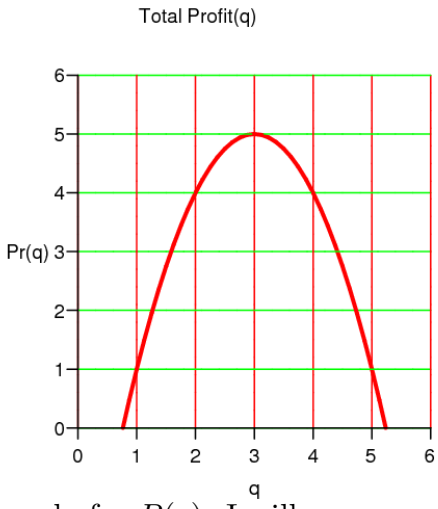
To view animations (work in progress):
`http://www2.math.uic.edu/~lewis/math165/165avgprofit.htm`.

Marginal Analysis Criterion for Maximal Average Profit

Closely related to *Marginal Analysis for Minimum Average Cost*. Not quite the same as *Marginal Analysis Criterion for Maximum Profit*. Hoffmann/Bradley, p. 242

Let $P(q)$ is the total profit of producing the first q units.

Here is the graph of a typical $P(q)$.



I won't tell you a specific formula for $P(q)$. I will assume:

- The graph of $P(q)$ is smooth and concave downward.
- $P(q) = 0$ has exactly two positive roots, the smallest is called the *break even point*.

The *average profit per unit*, $\text{Avg}P(q)$, of producing the first q units, is

$$\text{Avg}P(q) = P(q)/q.$$

Marginal Analysis Criterion for Maximal Average Profit. *Average profit per unit is maximized at the level of production where the average profit per unit equals the marginal profit; that is*

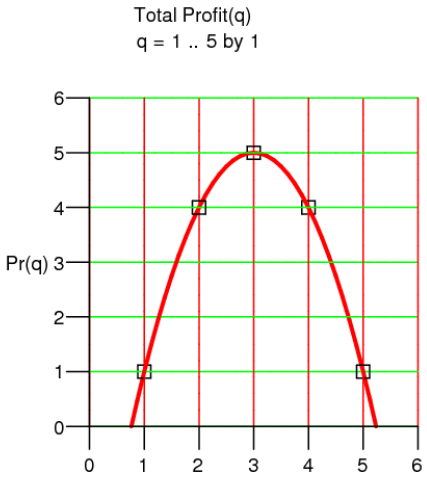
$$\text{Avg}P(q) = \frac{dP}{dq}.$$

Th proof is the quotient rule for differentiation of $P(q)/q$.

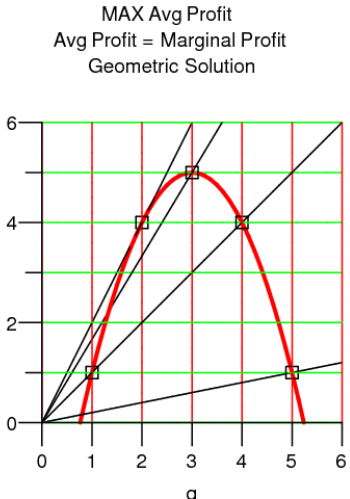
Here is a graphical explanation of this criterion:

The *average profit per unit* at q is the slope of the line from the origin 0 to the point $(q, P(q))$.

Look at the graph for various values of q,

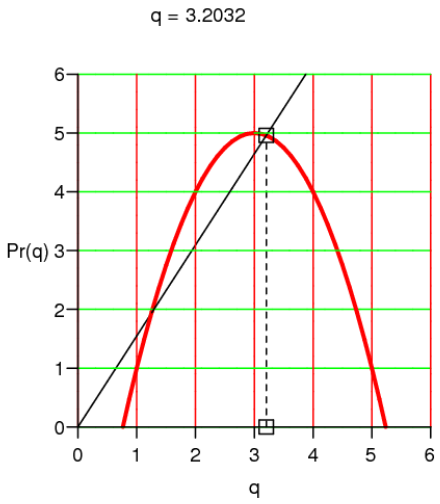


Use a straight edge or ruler to represent these lines.



As you move q to the right, the slope of the line from 0 to $(q, P(q))$ increases and then decreases. The maximum slope occurs when $q \approx 2$. At $q \approx 2$, the line from 0 to $(q, P(q))$ is tangent to the graph at $(q, P(q))$.

To view an animated picture (work in progress) go to:
`http://www2.math.uic.edu/~lewis/math165/165avgprofit.htm`.



Note that the condition

$$\frac{P(q)}{q} = \frac{dP}{dq}$$

is the same as

$$1 = \frac{q}{P}\frac{dP}{dq}.$$

The quantity $P_E = \dfrac{q}{P}\dfrac{dP}{dq}$ is the *elasticity of profit with respect to output* or *output elasticity of profit*.