MATERIALS FOR: LOGIC ACROSS THE HIGH SCHOOL CURRICULUM

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I have discussed in [Bal0x] the general outline of logical topics that are especially relevant for secondary teaching. Here I list some specific materials, most already used with teachers or pre-service, that tie these logical ideas to the high school classroom.

My main point is to connect mathematical problems that arise in high school with justifications that require 'proof'.

Algebra

Algebra examples include the juicy-juice problem in Subsection 1.1, problems on estimating size of fractions and similar in Gelfand's algebra, negative-times-negative is positive, integral domain and solution of quadratics, etc. etc.

Geometry

Geometry examples include: clarifying the redundancy of axioms in most geometry texts; discussing the differences among 'mathematician's proof', two-column proof, two-column proof done right (i.e. not like most extant books), and formal proof; examining in detail a basic notion like right angle.

One possibility would be to go into the differences between the axiomatization of geometry by Moise-Birkhoff, Hilbert, and Euclid. That might be too detailed. But, in any case, the understanding geometry in Hilbert's mode (tables, chairs, beer bottles) is central.

trigonometry and calculus

The first assignment in Section 1.1 illustrates confusion about parameters, variables, and quantifiers in a typical trig book.

Notions of limit and especially uniform versus pointwise continuity show quantifiers are important.

1. References

I have organized these materials by the class for which they were assigned.

1.1. **Math 215 Spring 07.** `http://www.math.uic.edu/~jbaldwin/math215`

first assignment: `http://www.math.uic.edu/~jbaldwin/math215/assign1a.pdf`

Note that this quote from a precalc book contains a hidden existential quantifier. The solution is at

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http://www.math.uic.edu/~jbaldwin/math215/son1a.pdf.

2nd assignment: builds from How juicy is it? (Connected Math middle school curriculum) to solving of inequalities.

http://www.math.uic.edu/%7Ejbaldwin/math215/assign2.pdf

The piece: 'Understanding two quantifiers' is both funny and germane.

http://www.math.uic.edu/~jbaldwin/math215/astell.pdf

1.2. **Algebra Initiative.** Notes and assignments from the course for 8th grade teachers in algebra are at

http://www.math.uic.edu/~jbaldwin/alginit/index

In the notes for Feb.17 through March 4 are a continuing series of examples and explanations that discuss the meaning of the disjunction and recognizing that the solution of an absolute value inequality is a derivation. I thought page 3 of the printable version of the March 4 lecture made the distinction between procedural and conceptual solution particularly clear. For secondary teachers the assignments can focus more on the underlying logic. But the connection with classroom teaching should be made in the same way.

1.3. **Barwise-Etchemendy.** A current webpage explaining what is available:

http://ggww2.stanford.edu/GUS/openproof/#LOFOL

Tarski's world only includes the semantics - $40 new from Amazon -but cheap copies available.

Language Proof and Logic includes a treatment of proof (and software for checking proofs. $56 Again, some cheaper quantities are available.

Grade grinder - computer checking of solutions for students who buy the book new.

I have used this both as a stand-alone reading course for high school teachers and as an introduction to a geometry course. It is essential to build exercises to bridge from the extremely concrete examples in Tarski's world to the use of quantifiers in everyday mathematics, such as Assignment 1 in the Math 215 course above.

1.4. **Mtht 400: Fall 2004.** http://www2.math.uic.edu/~jbaldwin/mtht400/index

The first assignment speed (with several reading assignments), and the assignments 'faulty reasoning', and 'reasoning and vocabulary' are particularly relevant. Faulty reasoning illustrates the result of overgeneralizing the substitution properties of equality.

1.5. **Math 592: Geometry and Proof.** I gave this as a graduate seminar. The course tried to understand the impact of the change in axiom systems for geometry that occurred under the new math on the teaching of geometry in the last 40 years. This is a good topic but there needs to be considerable background for understanding it.

The summary http://www2.math.uic.edu/~jbaldwin/math592/sum1117.pdf

tells what I tried to do in the course.

The assignment list is at

http://www2.math.uic.edu/~jbaldwin/math592/

The assignments from Tarski's world went through the beginning of Chapter 7 (with a lot of skipping) and were completed by Oct. 1.

Assignments 2 and 3 are an example of looking directly at a high school text and analyzing the difficulties in a particular formulation of geometric axioms in that curriculum. This comes down to such basic questions as. What is a right angle?

1.6. **Math Forum.** Here is an example of work on PEMDAS and FOIL. These two acronym are fundamentally linked to logical notions: parsing formal expressions and justifying manipulations.

http://www2.math.uic.edu/~jbaldwin/pub/tucsonpap.pdf

1.7. **Articles.** Several articles on my website are designed to explain a 'logical point of view to

mathematicians: http://www2.math.uic.edu/~jbaldwin/pub/birthbbltrev.pdf

and

mathematics educators:

http://www2.math.uic.edu/~jbaldwin/pub/var5.pdf

http://www2.math.uic.edu/~jbaldwin/pub/quitopaposu.pdf.

In particular the notes from lectures at the address below show how these issues have been addressed in talks for mathematics educators.

http://www2.math.uic.edu/~jbaldwin/mathed.html

These are not materials written for teachers although the second two contain many examples that could and have been used with teachers.

REFERENCES

[Bal0x] J.T. Baldwin. Logic across the high school curriculum. internal document, 200x.

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