Geometry, the Common Core, and Proof

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Chicago Teacher Transformation Institutes

1. MSP with 5 universities and CPS
2. Workshops on leadership, teaching and content
3. 4 course sequences in Math and in Science

CTTI `http://www.uic.edu/orgs/ctti/`

Workshop: `http://homepages.math.uic.edu/~jbaldwin/CTTIgeometry/ctti`
25 page text; 20 activities, slides from each week, links to materials, notes of sessions

Geometry Workshop

Teachers were extremely enthusiastic.

15-20 teachers each of 5 days; 25 distinct participants; they were paid.

Are workshops better than courses?

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A principle of professional development

A Focus on material that could occur in the classroom.

B But, present it in unexpected ways, to deepen understanding and connections.

The teacher needs to be a learner.

A. high school material

1. Common notions and Euclid's first 3 postulates in modern language
2. Basic constructions: (equilateral triangle, rusty compass)
3. Basic congruence axiom SSS and corollaries (SAS, ASA etc)
4. Parallel postulate and Parallelograms
5. Area and area formulas
6. The Golden ratio and irrational numbers
7. The side-splitter theorem and similarity

B. New Perspective

1. Big picture: 2 driving problems
2. Historical perspective: numbers through geometry
3. Focus on student understanding of concepts
4. Proof as a means of understanding and organizing
5. Axiom systems as implicit definitions of undefined terms.
6. Choice of axiom system has consequences
7. Connections with CCSSM

Day 1 Agenda

1. G-C0-1 – Context.
2. Activity: Divide a line into n pieces -with string, paper folding, using lined paper; via construction
3. Reflection activity (geometry/ proof/definition/ common core)
4. mini-lecture Axioms and Definitions in Euclid
5. Activity Construct an equilateral triangle. Prop 1.
6. Diagrams and proofs
7. lunch/Discussion: How do these differ from axioms in high school texts
8. Activity - rusty compass theorem (30 min)
9. congruence as a basic notion; SSS
10. Discussion of G-C0 1

Common Core

G-C01

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Why is the word undefined in this standard? What and how do we know about the undefined notions?

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G-C01

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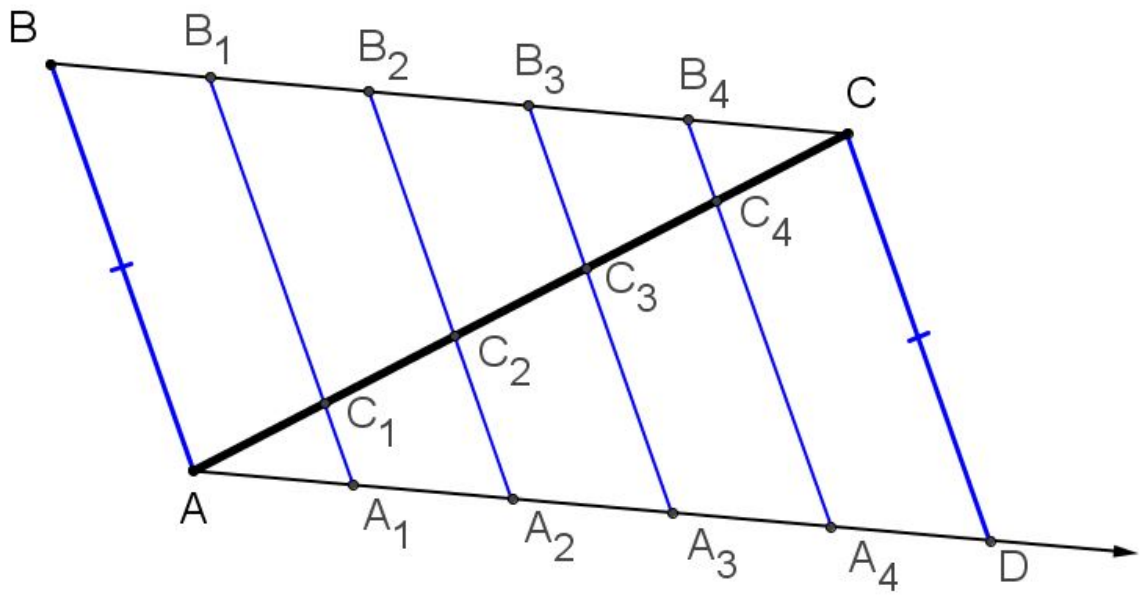
Why is the word undefined in this standard? What and how do we know about the undefined notions?

Why are both 'distance along a line' and 'distance around a circular arc' in the list of undefined concepts instead of just 'distance'?

Activity: Dividing a line into n-parts: Diagram



Why did it work?

Why can you make the construction?

Why does it work?

Why did it work?

Why can you make the construction?

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Why are the lines parallel?

Why did it work?

Why can you make the construction?

Why does it work?

Why are the lines parallel?

Why is that enough?

Common Notions

Common notion 1. Things which equal the same thing also equal one another.
Common notion 2. If equals are added to equals, then the wholes are equal.
Common notion 3. If equals are subtracted from equals, then the remainders are equal.
Common notion 4. Things which coincide with one another equal one another.
Common notion 5. The whole is greater than the part.

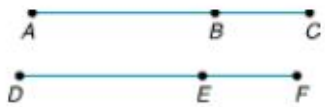
Modern (new math) text books make a big deal about the difference between congruence and equality. Numbers are central - so equalities are only between numbers while line segments or figures are congruent.

The Cure is worse than the disease ?!

Proof of CN 3 for segments from CN3 for real numbers

Proving Segment Relationships

1. For the proof shown, provide statement 2.



Given: $\overline{AC} \cong \overline{DF}$
$\overline{BC} \cong \overline{EF}$

Prove: $\overline{AB} \cong \overline{DE}$

Proof:

Statements	Reasons
1. $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$	a. Given.
2. __?__	b. Definition of congruent segments
3. $AC - BC = DF - EF$	c. __?__
4. $AC - BC = AB$, $DF - EF = DE$	d. Segment Addition Postulate
5. __?__	e. Substitution Property (=)
6. $\overline{AB} \cong \overline{DE}$	f. __?__

A. $AB = BC, DE = EF$

B. $AC = DF, BC = EF$

C. $AC = DE, AB = DF$

D. $AB = DE$

Question

Why does it take six steps to show: If two line segments have the same length and equal line segments are taken away from each, the resulting segments have the same length.

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Following Euclid: In the common notions, Equality can mean: identity, natural number equality, segment congruence, same area, figure congruence

Geometry before Number

Euclid did not have lengths as distinct objects - He'd say line segments are congruent or equal where we'd say "have the same length".

The difference between counting numbers and measuring numbers became clear in the workshop.

Our Axiomatic Approach

Synthesizing Euclid and Hilbert

1. Euclid's construction axioms
2. All right angles are equal
3. SSS
4. Betweenness treated informally
5. Parallel Postulate
6. Area axioms

3 weaknesses in Euclid (for high school)

1. Superposition (fixed by assuming SSS)
2. Vagueness of diagrams (Not a problem for 2000 years)
3. theory of limits (fixed by segment arithmetic)

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The two driving problems

30 hours of workshop time aimed at two problems:

1. Prove the method of dividing a line into n equal segments works.
2. What if the segments are not equal or not even commensurable?

Dividing the line into equal segments

For the first task we must:

1. do and justify basic constructions
2. understand congruent triangles SAS, SSS
3. study parallel postulate and comparing angles.

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Gave Varignon's theorem as example but it morphed into a proof.

What if the segments are not equal?

Irrational Numbers

What if the ratios are irrational?

What does that mean? geometric proof of the irrationality of the golden ratio.

Theorem

Side-splitter Theorem : Euclid VI.2 **CCSS G-SRT.4**

A line cutting a triangle is parallel to a side iff it cuts the other two sides proportionally.

Proof of Side-splitter Theorem

Without using limits, we proved the full side splitter theorem rigorously.

Ingredients

1. Segment Arithmetic;
2. Area axioms: proof that area is computed in segment arithmetic by the usual formulas;
3. Proof of side-splitter as in CME geometry.

CCSM geometry standards

The following items don't appear in the CCSS high school geometry standards.

1. parallel postulate,
2. area of polygons,
3. irrational numbers