10/13/2012
CTTI Geometry workshop notes

Intro activity (pipe cleaners) group comments: (appx 9am-10am)

-Given an arbitrary length, extension(multiplication) is easy. (Sub)division is hard.

-Subdivision strategy: Construct similar triangles (proof sketch; incomplete)

-Fold paper into square and draw diagonals; use intersections in some way

Initial discussion: (10am-11:30am)
-How do we use geometry?: drawing designs, engineering/blueprints; 2d/3d, construction strength/efficiency; logical-deductive thinking; exploration/design.

-Axioms interpreted as "rules of the game"; theorems as the "plays you make" according to the rules.

-Discussion of 19th C. foundational crisis touched off by non-Euclidean geo; multiple models.

-What elementary principles do we use in Geometry? Do we use only the axioms, or do we employ the axioms ALONG WITH other basic logical notions?

-Geometric constructions and reasoning—how useful is this to students? Helpful for student to see a full proof and walk through the steps; Engages visual/tactile senses by using modeling & hands-on tools.

-Problem raised: Use of tools introduces human error, even with "more accurate" straightedge and compass constructions.

-Comment: Geometric construction was necessary for pre-GPS navigational tools.

-Hon Fong alternate solution of segment subdivision problem: Employed translation of triangle.

-Role of hypothesis: To determine when you have a meaningful statement?

-Hypothesis in math vs. hypothesis in science: math—assumptions you bring to a problem

Geo since 1970: making a distinction between "congruence" and numerical equality
-real number length introduced through the 'back door'
-"equal"(abstract numerical) vs. "congruent"(phys. coindence)
- meaning of "is"; identity vs. congruence. Is one segment "the same" as another that is congruent to it?

Pre-lunch discussion: (11:30-12:30)

-Constructing an equilateral triangle: asked for 3 solutions, one was to construct sqrt(3) and then use it as base of right triangle (i.e., make a 30-60-90 triangle and reflect into 60-60-60)

-Inscribe hexagon in a circle: Question—why does this work?

Post-Lunch discussion (1:30-3:00)

Andreas's section:
-Discussion of whether students can follow a given argument vs. being able to construct one on their own.

G-C0 1 activity
-[Add "between" to words they can use in definition]

Discussion of CPCTC:

How do we define "angle"?
-proposals:
-Angle as rotation of a ray
-Do we count the space in between the rays or not?
-Do we include the ray boundaries?
-Angle = "the figure formed by two line segments sharing an individual point"
    -problem: internal or external angle? If we choose the "smaller" one, what does "smaller" mean?
    -Use notion of "convexity" to define the rays AND the interior.

-Euclid and the omission of "straight angles" and obtuse angles
    -Not necessary for constructions.
    -But we need them later for evaluating trig functions with larger inputs.

Proposed definition: Right Angle = ¼ of full rotation-Len
-two together make a straight line
-Need to define "together"/"adjacent", but this is coherently interpretable from visual diagram
-Alt. definition: Two of the angles of intersecting lines, when all angles are equal
    -Omits necessity of defining "together"/"adjacent"

Other methods of angle measure? Cf. grads.

Summary: 4 units suggested: turns, radians, degrees, grads. See wikipedia

Proposed definition: Perpendicular Lines: comes easily from "right angle" definition

SSS—ensures translational regularity of the plane.

Define Arc: need "betweenness"—fine to assume meaning of "between" from visual diagram

-Hilbert introduced careful axioms for betweenness to avoid the difficulties with reading diagrams.

Proposed definition: Parallel Lines = lines that do not meet

Other proposals:
-lines that are equidistant, i.e., constant *perpendicular* distance

-student difficulties: is Perpendicular distance understood? How about parallel lines and railroad tracks? Do you need to "see" the lines intersect in order to establish that they are not parallel?

-Do students have trouble with the notion of extending lines "indefinite", i.e., beyond visual range?

-other useful physical models? 3d skew lines, not parallel. Implicit assumption of same plane

Definition according to circular arc: cf. radian measure

-using arc length instead of angle degree prepares for the subsequent introduction of radian measure in trig functions.

-Question: How should we assign measure to the angle with ray length one subtending an arc?

-ratio of arc length to circumference

-issue of whether or not you must use transcendental numbers

(Added in proof: The axioms to get arc length `commensurable with segment length' are significantly stronger (implying transcendental numbers). That is why both are undefined terms in the Common Core G-C01.