# MCS471 — Spring99 — Numerical Analysis — Hanson

## FINAL EXAMINATION

1pm to 3pm — 04 May 99 — 316 Burnham Hall

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**THE RULES**:

*A. Receiving or giving aid in a final examination is a cause for dismissal from the University.*
*B. Keep your eyes on your own exam booklet and keep your own exam booklet covered.*
*C. You may use a double-sided 8.5”×11” **Formula Sheet**, but on other paper besides exam booklets and you must hand in your signed formula sheet even if otherwise blank (no examples and problem can be on the sheet).*
*D. In computational questions, use 4 Digit Exam Precision: Chop to 4 significant digits only when you record an intermediate or final answer in your exam booklet; and continue calculations with these chopped, recorded numbers.*

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0. State your calculator’s a) Brand Name (e.g., TI?) b) Model Number (e.g., 81?) c) Displayed Digits (e.g., 10?) d) Is your calculator Programmable?

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1. Use an **efficient, composite, 5-point Trapezoidal Rule** to numerically approximate the integral of a piecewise function given by

$$\int_{0.5}^{1.5} f(x)dx, \quad \text{where} \quad f(x) = \left\{ \begin{array}{ll} \frac{x^2+3}{4}, & 0.5 \le x \le 1.0 \\ \frac{x+4}{x^2+4}, & 1.0 < x \le 1.5 \end{array} \right\},$$

minimizing the number of floating point operations and function evaluations. Also, tabulate

| $i$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $x_i$ | | | | | |
| $f_i$ | | | | | |

for all values used. {*Caution: You must treat* $f(x)$ *as one continuous function.*} (40)

2. Use a **composite, 1-point Gaussian Quadrature** ($3 \times G_1$ ; i.e., 3 points total) to numerically approximate the integral given by

$$\int_a^b f(x)dx = \int_0^{0.75} \frac{e^{2x}}{x+8}dx,$$

minimizing the number of floating point operations and function evaluations. Also, tabulate

| $i$ | 1 | 2 | 3 |
|---|---|---|---|
| $t_i$ | | | |
| $x_i$ | | | |
| $F_i$ | | | |

for all values used, where $F_i = F(t_i)$. (40)

*Hello, Turn Over for Page 2* ⟶

3. **Estimate the maximum step size $h$ for Euler's Method** applied to the first order ODE

$$y'(x) = f(x, y(x))$$

needed to make the absolute value of the theoretical global discretization error less than $tol = 0.45e - 2$, given the $(x, y)$-dependent bounds

$$|y| \leq 0.5223, \quad |f(x,y)| \leq 0.5649,$$
$$|f_x(x,y)| \leq 0.1432|x| \cdot |y|, \quad |f_y(x,y)| \leq 0.04321|x|^2,$$

for each $x$ on $[-0.5710, +0.4710]$. (40)

4. Using the **4th Order Runge-Kutta Method**, numerically approximate the solution to the nonlinear IVP by

$$y'(x) = 0.4710 \cdot y(x) \cdot (1 - 0.05444 \cdot y(x)/(1 + x^2)), \;\; y(1) = 200,$$

with $h = 0.025$ for two $x$-steps. Tabulate

| $n$ | $X_n$ | $Y_n$ | RK1 | RK2 | RK3 | RK4 | $X_{n+1}$ | $Y_n + \Delta Y_n$ |
|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | |
| 1 | | | | | | | | |

for all values calculated. (40)

5. Using the **Algebraic BVP Method**, numerically approximate the solution to the linear BVP,

$$y''(x) + x \cdot y'(x) \; + \; 3 \cdot y(x) = -0.2 \cdot x^2,$$
$$y(1.25) = 2.5 \;\; \& \;\; y(2.00) = 1.5,$$

using finite central differencing of derivatives with $h = 0.25$. Tabulate

| $n$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $X_n$ | | | | |
| $Y_n$ | | | | |

In solving, use the Thomas tridiagonal elimination algorithm. Sketch your approximation $Y_n$ in the $xy$-plane. (40)