

Last week, I. Kapovich and Weidmann posted a paper showing that there is an algorithm to solve the rank problem for torsion free, convex cocompact Kleinian groups. I haven't read enough of their paper to understand how it goes, but from what I can gather, they use generalizations of Nielsen techniques to actions on word-hyperbolic spaces, which they have developed in previous papers. They also make use of tameness of subgroups. It would be interesting to generalize their result to show that there is an algorithm to compute the rank for all fundamental groups of (good) compact 3-orbifolds.

Boileau and Weidmann have a paper which gives a structure theorem for 3-manifolds with non-trivial JSJ decomposition and which have two-generated fundamental group. They would like to rule out one case, which comes down to showing that a 2-bridge link cannot be an irregular cover of a manifold with a single boundary torus. I believe it should be possible to show this using the techniques I've developed for classifying 2-parabolic generator groups. In fact, I think one should be able to classify the commensurators of (non-arithmetic) 2-bridge links, and then show that there is no manifold cover between a 2-bridge link and the minimal orbifold which it covers (one may deal with the arithmetic case similarly). The idea is to take an orbifold quotient of the commensurator which turns the 2-generator group into a dihedral group. Then one would have to do a case by case analysis of dihedral subgroups of spherical 3-orbifolds. I believe it should be possible to generalize these techniques to classify orbifolds which have a fundamental group generated by two elements whose commutator is elliptic or parabolic (this generalization was suggested to me by Sakuma). One may also be able to show subgroup separability of such groups and classify generating pairs using similar tricks.

I chatted with Benson Farb last Friday, who mentioned several questions he is interested in. He, Alperin, and Noskov found a method to show that a 2 generator group acting on a singular $CAT(0)$ space

is free, which implies a strong Schottky type condition for the action. It is known that the reduced Burau representation on B_4 is faithful if and only if it is faithful on a particular 2-generator free subgroup of B_4 . They then consider the action of $B_4 \rightarrow \mathrm{GL}_3(\mathbb{Z}[t, t^{-1}])$ on the affine building associated to $\mathrm{GL}_3(\mathbb{Q}(t))$. But unfortunately their condition for freeness fails on this 2-generator subgroup. They do however show that the representation restricted to this subgroup may be deformed infinitesimally to faithful representations by using their criterion. I tried to determine for what values of t the subgroup could be conjugate in to $SO(2, 1)$, but it turns out that none of them work. But if t is chosen to be a complex number of norm 1, then the Burau representation is unitary, and I believe it lies in $SU(2, 1)$ if t is chosen correctly. If t is a cyclotomic number of order p , then the representation is not faithful on the subgroup, but it may have image $\mathbb{Z}_p * \mathbb{Z}_p$. In any case, one should be able to embed this image in a higher rank lattice, and maybe apply Schottky type techniques to show it is a free product for all p .