RESEARCH BLOG 11/17/03

I did a MathSciNet search to see if there were any papers which studied boundaries of convex cores of negatively curved spaces. After a fruitless search, I did a Google search and eventually found notes of Ritoré which discuss a proof of Bruce Kleiner [1] of the optimal isoperimetric inequality for pinched negatively curved complete simply connected 3-manifolds M. If the sectional curvature satisfies $K(M) \leq -1$, then Kleiner shows that a smooth sphere in M bounds a volume less than that bounded by a round sphere in hyperbolic space. This answered a conjecture of Aubin and others. Anyway, in his argument, he first proves that a region maximizing volume for a given area must be convex, by taking the convex hull of a candidate sphere maximizing volume. He needs to understand the geometry of the boundary of a convex hull, and he uses a technique of Almgren by approximating by the distance spheres of radius r, letting $r \to 0$. Anyway, I think his technique should work in the case of convex cores of geometrically finite manifolds with pinched negative curvature, but I haven't worked this out yet.

I claimed in the last blog that I could generalize an argument I made in a special case before for end-irreducible manifolds. The setup is that we have a manifold M with pinched negative curvature, and a geodesic g' such that $M - g'$ also has a metric with pinched negative curvature. Moreover, the end-reduction V at g' has the property that $\pi_1(V) = \pi_1(M)$ and $\pi_1(V - g') = \pi_1(M - g')$. In this case, we want to prove that M is a geometric limit of geometrically finite pinched negative curvature manifolds which are homotopy equivalent. We use a similar strategy to blog 11/13/03.

Let $g' \subset M_i$ be an exhaustion of V by regular compact submanifolds such that ∂M_i is incompressible in $M - g'$. Since $\pi_1(V) = \pi_1(M)$, there is a compact core $C \subset V$ for M (and for V!), which we may assume lies in M_1. Let M_i' be the cover of $M - g'$ with $\pi_1(M_i') = \pi_1(M_i - g')$. Then $M_i - g'$ lifts to M_i', and filling in g' we get M_i lifts to $M_i'(g')$. In particular

C lifts to $M_i'(g')$. Take the cover $N_i \to M_i'(g')$ corresponding to $\pi_1(C)$. M_i' is tame (by Canary's covering theorem generalized to negatively curved context), and therefore $M_i'(g')$ is tame, so N_i is tame.

Choose $g' \subset K \subset M$, K compact. Then $\pi_1(K - g') \subset \pi_1(M_i - g')$ for some i. So by covering lifting, $K - g' \hookrightarrow M_i'$, and therefore $K \hookrightarrow M_i'(g')$, and thus we have a map $\pi_1(K) \to \pi_1(M_i)$, since $M_i'(g')$ is tame (and thus a natural map $\pi_1(K) \to \pi_1(M_j)$). In fact, the composition $K \hookrightarrow M_i'(g') \to M$ is the inclusion map. Now, an argument given before shows that $ker[\pi_1(M_i) \to \pi_1(M)] \subset ker[\pi_1(M_i) \to \pi_1(M_j)]$ for some j. So

$$ker[\pi_1(K) \to \pi_1(M)] \subset ker[\pi_1(K) \to \pi_1(M_j)].$$

This means that we have a lifting $K \hookrightarrow N_j$. Thus, M is a geometric limit of the sequence N_i.

References

[1] B. Kleiner. An isoperimetric comparison theorem. *Invent. Math.*, 108(1):37–47, 1992.