RESEARCH BLOG 11/14/03

I claimed last time that I wanted to reduce the general case of Marden's conjecture to the special case when $M - g'$ may be exhausted by incompressible surfaces. I don't know how to do this, but I think one may be able to get around some difficulties by implementing techniques of Canary, Minsky, and Souto directly.

First, I need to recap some of the ideas of Bob Myers adapted to the context I set up in the last blog 11/13/03. A year ago, I came up with some similar ideas by thinking about minimal surface representatives of incompressible surfaces, but Myers' approach is simpler, and takes advantage of technology already in the literature. What Myers proves is that there is an open submanifold $g' \subset V \subset M$, such that V is an *end reduction of* M *at* g' (this notion appears to be due to Brin and Thickstun, who prove that it is unique up to non-ambient isotopy). This means that $V - g'$ is irreducible, $M - V$ has no components with compact closure, V has an exhaustion by compact submanifolds containing g' whose boundary is incompressible in $V - g'$, and any incompressible surface in $M - g'$ can be isotoped in $M - g'$ to lie in $V - g'$ (so V engulfs all the incompressible surfaces in $M - g'$). The relevant fact that Myers shows is that $\pi_1(V) \cong \pi_1(M)$. It is also clear that $\pi_1(V - g')$ injects into $\pi_1(M - g')$.

Myers' goal was to prove a topological conjecture of Freedman which would imply the Marden conjecture, but we would like to make more use of the negative curvature assumption. What Myers wanted was that $\pi_1(V - g') \cong \pi_1(M - g')$. We can get this condition by passing to a cover $N \to M - g'$ such that $\pi_1(N) = \pi_1(V - g')$ (this is very similar to the trick we used last blog). Then $V - g'$ lifts to N, and as last time, we may fill g' back in to get a negatively curved manifold $N(g')$ and an isometric embedding $V \to N(g')$, which one can show is also and end reduction of $N(g')$ at g'. Unfortunately, we do not know that $N(g') \cong V$, otherwise we could reduce to the case considered last time. But assuming that we can show that $N(g')$ is tame, let's see how to finish the argument. In this case, V is tame, so this means that V may be taken to be the interior of a compact core $g' \subset C \subset M$. But then ∂C is incompressible in $M - g'$. What we've just shown is that every algebraically diskbusting geodesic, perturbed to be an embedded link l in M, is contained in a compact core C of M. If we assume that g' is a sublink of l, then I think we may assume that ∂C is constant up to isotopy in $M - g'$, using an argument of Souto (although I haven't gone through his proof to check that the details carry over to our context). A result of Bonahon [1] implies that there are geodesics exiting any geometrically infinite end of M. Take such an embedded geodesic h, which is very far from g'. Then we may isotope ∂C in $M - g'$ so that it contains h. Thus, we may lift $h \cup g'$ to $N(g')$. But this means that the end of $N(g')$ has geodesics exiting it, so by Canary's theorem [2], $N(g')$ should be geometrically infinite. Thus, by Canary's covering theorem [3], $M = N(g')$, so M is tame.

So let us review the context in which we need to prove tameness: we have an algebraically diskbusting curve g' in a negatively curved manifold M, such that $g' \subset V \subset M$ is an end-reduction with $\pi_1(V - g') \cong \pi_1(M - g')$. I think that the arguments of Canary and Minsky [4] as modified by Souto [5] may still carry over to this case. The main difficulty seems to be to deal with simplicial ruled surfaces and boundaries of convex cores in pinched negative curvature. What we want to show is that there are simplicial surfaces homotopic to the boundary of a compact core in $M - g'$ exiting each end. But I need to understand their arguments better before I can see if they carry over to the context I am considering.

References

[1] F. Bonahon. Bouts des variétés hyperboliques de dimension 3. *Ann. of Math. (2)*, 124(1):71–158, 1986.
[2] R. D. Canary. Ends of hyperbolic 3-manifolds. *J. Amer. Math. Soc.*, 6(1):1–35, 1993.
[3] R. D. Canary. A covering theorem for hyperbolic 3-manifolds and its applications. *Topology*, 35(3):751–778, 1996.
[4] R. D. Canary and Y. N. Minsky. On limits of tame hyperbolic 3-manifolds. *J. Differential Geom.*, 43(1):1–41, 1996.
[5] J. Souto. A note on the tameness of hyperbolic 3-manifolds, 2002. preprint http://www.math.uni-bonn.de/people/souto/compact-cores.ps.