RESEARCH BLOG 8/18/03

I spent two weeks in Cambridge at the Isaac Newton Institute at the program on Spaces of Kleinian groups and Hyperbolic 3-manifolds. Notes from the talks will eventually be available at the web site, I'll mention in this blog when this occurs.

One of the main goals of the field of Kleinian groups is to give a complete classification of them. Another major goal is to understand qualitative and quantitative geometric and dynamical information about Kleinian groups. There has been much progress on these questions, and the conference had many of the experts there (some people were missing for personal reasons, such as Jeff Brock who just had a new baby). Thurston proposed a classification of Kleinian groups called the ending lamination conjecture, which gives a parameterization in terms of the structure of the hyperbolic manifold at infinity. Brock, Canary, and Minsky announced the proof of this conjecture for the class of tame hyperbolic manifolds, completing a program which Minsky has developed over many years (and depending on work of many others, especially Masur). The question of whether hyperbolic manifolds with finitely generated fundamental groups are tame is known as the Marden conjecture, and seems to be the main obstruction to the full ending lamination conjecture. Brock and Souto announced that the have proved this in the case of manifolds which are limits of tame manifolds, completing a series of work by many people starting with cases proved by Thurston. Ken Bromberg announced a proof of Bers' density conjecture, and gave a conjecture that would imply the ending lamination conjecture using techniques completely independent of Minsky's program (based on work of Hodgson and Kerckhoff). Ken's conjecture states that if one considers geometrically finite surface groups in a Bers slice, then the space of structures with parabolics pinched on one end is bounded in a certain metric coming from quadratic differentials. He uses a trick which involves subgroup separability of surface groups, to get a finite sheeted cover which has a closed geodesic with a large tube around it, which he may then use a drilling trick which he used to prove density. Richard Evans discussed a special case of this approach, when one assumes "super slender" (?) geometry (sequences of maximal collections of curves on a surface are very short, in which case they may be drilled, and then a unique limit extracted). The ending lamination conjecture does not give a complete picture of the space of Kleinian groups: it gives a continuous parameterization from ending data to Kleinian groups, but this might not be 1-1. To give a 1-1 parameterization, one must understand the phenomenon of "bumping", where two components of the interior have closures which intersect (where an accidental parabolic element occurs). John Holt discussed this, where he gives geometrical conditions under which bumping cannot occur. Later, I'll try to discuss other talks from the conference which were focussed on the geometry of Kleinian groups, rather than their classification.

Of course, for finite volume hyperbolic manifolds, a complete classification would be given by the geometrization conjecture, which would imply that these are in 1-1 correspondence with (homotopically) atoroidal, irreducible 3-manifolds with infinite fundamental group. I've made a bit more progress in understanding Perelman's work on proving geometrization, which I'll discuss next time.