Notes on Perelman's work are now available. Kleiner and Lott have written up notes on Perelman's first paper. Perelman's papers omit many computations and details, so hopefully their notes will make it much easier to read. There is also a link to notes on Perelman's lectures at Stonybrook by Christine Sormani, which should shed some light on his program. I've added slides on my talk on conjectures on minimal surfaces and Ricci flow, which was given at Caltech, UCSB, and the Georgia topology conference.

A paper was posted generalizing the volume conjecture by Gukov. This conjecture relate limits of colored Jones polynomials to the volume of Dehn fillings on a hyperbolic knot. In particular, the conjecture would imply that the colored Jones polynomials completely determine the A-polynomial of a hyperbolic knot. This seems to be related to work of Gelca, Frohman, and Lofaro, who generalize the A-polynomial to a non-commutative version, and implicitly conjecture that it determines the colored Jones polynomials. I suppose one could conjecture that the colored Jones polynomials also determine the non-commutative A-polynomial. If one believes Gukov's conjecture for all knots (not just hyperbolic), then it would seem that the generalization of the volume conjecture to non-hyperbolic knots should really give the maximal volume of a representation associated to a knot, rather than the simplicial volume. It would be interesting to consider the difference

$$\lim_{N\to\infty} \log(|J_N(K, e^{2\pi i/N})| - [\frac{1}{2\pi} Vol(M)]^N)/N.$$

I would conjecture that this converges to something related to the second largest volume of a representation of a knot complement (this should be 0 for the figure 8 knot complement, but should be non-zero for other examples). This conjecture is motivated by the analogy with finding eigenvalues of a matrix: give an $n \times n$ matrix A, the largest eigenvalue is given by $\lambda_1 = \lim_{n\to\infty} \frac{1}{n} \log \mathrm{tr}(A^n)$, and the second largest eigenvalue is $\lambda_2 = \lim_{n\to\infty} \frac{1}{n}\log(\mathrm{tr}(A^n) - \lambda_1^n)$. There might be analogous conjectures generalizing the conjectures of Gukov to finding volumes of representations of Dehn fillings of a knot complement.

The Georgia topology conference was held last week, hosted by Nancy Wrinkle. Tara Brendle discussed generating the mapping class group by torsion elements. Wajnryb showed that mapping class groups of closed surfaces may be generated by two elements. Brendle and Farb show that they may be generated by three torsion elements (one of which may be taken to be an involution) or seven involutions. I think it might be possible to generate by five involutions, if one could show that any torsion element is a product of two involutions. I conjecture that any torsion element in the mapping class group lies in a dihedral subgroup, so is therefore a product of involutions (I think Brendle and Farb actually only consider the orientation preserving mapping class group, otherwise I think it would be easy to show that a torsion element is a product of two reflections).

John Etnyre discussed new invariants of knots coming from contact topology. I'll briefly describe how this works for knots in $\mathbb{R}^3$. Consider the unit cotangent space $T_1^*(M)$ to a Riemannian manifold M^n, then this is a $2n-1$ dimensional manifold. T^*M has a tautological 1-form, such that for any 1-form $\alpha : M \to T^*M$, $\alpha^*\lambda = \alpha$. Then $\lambda|_{T_1^*M}$ is a contact form, that is $\lambda \wedge d\lambda^{n-1} \neq 0$. Given a transversely immersed map $f : N \to M$, one gets an associated embedded Legendrian submanifold of T_1^*M given by the unit conormal bundle to N. For example, for a knot $S^1 \in \mathbb{R}^3$, one gets a Legendrian torus (tangent to the contact planes) in $T_1^*\mathbb{R}^3$. The contact isotopy type of this torus is a (non-trivial) knot invariant. To see why this might be true, note that if one homotopes a knot to intersect itself, then there will be a unit 1-form vanishing on the tangent space spanned by the tangent vectors of the two strands going through the point of intersection, so the Legendrian torus develops a self-intersection. Ekholm , Etnyre, and M. Sullivan have developed a contact homology of Eliashberg and Hofer to get invariants of the contact tori, and therefore of the knots. Lenny Ng defined combinatorial invariants motivated by this construction, and was able to show they are well-defined without going through the analysis. Etnyre conjectures that the contact homology invariants they have defined coincide with Ng's invariants, but there are difficulties in showing that their homology satisfies Ng's combinatorial conditions. Josh Sabloff mentioned that physicists believe that these invariants are related to quantum invariants, such as the colored Jones polynomial, by some sort of string theory duality. There might also be connections with Floer homology invariants of Oszvath and Szabo.

Rob Kusner discussed an interesting conjecture, which I'll call the spaghetti conjecture. Take an 1-manifold in $\mathbb{R}^3$ with tube radius 1, and consider the asymptotic density of the radius 1 tube. Then the density is at most $\pi/\sqrt{12}$, the density of the hexagonal packing of disks in the plane. In other words, spaghetti packs best the way it is packaged!

Genevieve Walsh discussed the virtual Haken conjecture for 2-bridge knots. Since the 2-fold branched cover of a 2-bridge knot is a lens space, a dihedral covering gives a great-circle link in S^3. A Dehn filling of the knot will lift to the 2-fold cover (and therefore to the dihedral cover) if it is of slope $2p/q$. Genevieve finds π_1-injective surfaces in many examples of 2-bridge knots which lift to an embedding in the dihedral cover (if the 2-fold cover is the lens space $L(r,s)$, then she requires $r/s < \frac{1}{4}$), and she shows that the dihedral cover of the Dehn fillings of the form $2p/q$ will usually be Haken.

Noah Goodman discussed his proof of Harer's conjecture. In his thesis, he has written up a proof of Giroux's theorem that open book decompositions of a manifold up to positive plumbing with Hopf bands give equivalence classes of contact structures. His proof makes use of studying contact structures which are normalized with respect to a cell structure (whereas Giroux uses triangulations). Harer's conjecture states that any two fibered links in S^3 are connected by a sequence of plumbings with Hopf bands. He and Giroux independently noticed that Giroux's theorem implies Harer's conjecture. The idea is something like this: the plane fields of contact structures are classified up to homotopy by a $\mathbb{Z}$-valued invariant λ (which we'll associate to a fibered link). There are two Hopf bands, $H^{\pm}$, so that the contact structure associated to H^+ is tight and has $\lambda(H^+) = 0$, and $\lambda(H^-) = 1$. Given two fibered links L_1 and L_2, if $\lambda(L_1) \leq \lambda(L_2)$, then stabilize L_1 by H^- several times to get $\lambda(L_1 * H^- * \cdots * H^-) = \lambda(L_2)$, using $\lambda(L_1 * L_2) = \lambda(L_1) + \lambda(L_2)$, and $*$ denotes plumbing. If $\lambda(L_2) = 0$, stabilize both by H^- to get links with associated overtwisted contact plane fields. By a theorem of Eliashberg, two overtwisted contact plane fields are homotopic iff they are isotopic. Thus, by Giroux's theorem, they are equivalent by $*H^+$ stabilization.