

```
In[1]:=
(** The program enumerates the maximal trees
    in a graph by using the Wang algebra. Wang algebra is mod-
    2 polynomial algebra in a set of variables such that the square of
    each variable is equal to zero. To find the trees in a graph G,
    label each edge of the graph with a separate variable; choose all but one vertex;
    for each chosen vertex write the sum of the edge labels at that vertex;
    take the product of these sums in the Wang algebra. Each tree
    will appear once as a term in the expansion of this product. **)
```

```
rule1 = {X[a_, b_] => X[a] + X[b], X[a_, b_, c_] => X[a] + X[b] + X[c],
  X[a_, b_, c_, d_] => X[a] + X[b] + X[c] + X[d],
  X[a_, b_, c_, d_, e_] => X[a] + X[b] + X[c] + X[d] + X[e]}
rule2 = {X[a_] X[a_] => 0}
rule3 = {X[a_] => 1}
Prep[t_] := Expand[Simplify[(t /. rule1 // Expand) /. rule1]]
Tree[t_] := Expand[Simplify[(Prep[t] /. rule2 // Expand) /. rule2], Modulus -> 2]
Num[t_] := Expand[Simplify[(Tree[t] /. rule3 // Expand) /. rule3]]
```

```
Out[1]= {X[a_, b_] => X[a] + X[b], X[a_, b_, c_] => X[a] + X[b] + X[c],
  X[a_, b_, c_, d_] => X[a] + X[b] + X[c] + X[d],
  X[a_, b_, c_, d_, e_] => X[a] + X[b] + X[c] + X[d] + X[e]}
```

```
Out[2]= {X[a_]^2 => 0}
```

```
Out[3]= {X[a_] => 1}
```

General::spell1 : Possible spelling error: new symbol name "Tree" is similar to existing symbol "True".

```
In[7]:=
Num[X[a, b, f] X[b, c, g] X[c, d, h] X[d, i] X[e, j] X[j, f, k] X[k, g, l] X[l, h, m] X[m, i]]
```

```
Out[7]= 209
```

```
In[8]:=
Num[X[a, c] X[a, b, d] X[b, e] X[c, h, f] X[d, f, g, i] X[e, g, j] X[i, k, l] X[j, l]]
```

```
Out[8]= 192
```

```
In[9]:=
Tree[X[a, b] X[b, c]]
```

```
Out[9]= X[a] X[b] + X[a] X[c] + X[b] X[c]
```

```
In[10]:=
Num[X[a, b, c] X[b, g] X[c, e, f] X[d, e] X[f, g]]
```

```
Out[10]= 15
```

```
In[11]:=
Num[X[a, b] X[d, e] X[b, c, d]]
Tree[X[a, b] X[d, e] X[b, c, d]]
```

```
Out[11]= 8
```

```
Out[12]= X[a] X[b] X[d] + X[a] X[c] X[d] + X[b] X[c] X[d] + X[a] X[b] X[e] +
  X[a] X[c] X[e] + X[b] X[c] X[e] + X[a] X[d] X[e] + X[b] X[d] X[e]
```

In[13]:=

**Num[X[a, b, c] X[a, b, d]]**  
**Tree[X[a, b, c] X[a, b, d]]**

Out[13]= 5

Out[14]= X[a] X[c] + X[b] X[c] + X[a] X[d] + X[b] X[d] + X[c] X[d]

In[15]:=

**Num[X[1, 4, 6] X[5, 6, 7, 8] X[2, 3, 7] X[3, 4, 8]]**

Out[15]= 45

In[16]:=

**Num[X[1, 2] X[2, 3] X[3, 4] X[4, 5] X[5, 6] X[6]]**  
**Tree[X[1, 2] X[2, 3] X[3, 4] X[4, 5] X[5, 6] X[6]]**

Out[16]= 1

Out[17]= X[1] X[2] X[3] X[4] X[5] X[6]

In[18]:=

**Num[X[1, 2, 3] X[4, 5, 6] X[6, 7] X[1, 2, 5, 7]]**  
**Tree[X[1, 2, 3] X[4, 5, 6] X[6, 7] X[1, 2, 5, 7]]**

Out[18]= 19

Out[19]= X[1] X[3] X[4] X[6] + X[2] X[3] X[4] X[6] + X[1] X[3] X[5] X[6] +  
 X[2] X[3] X[5] X[6] + X[1] X[4] X[5] X[6] + X[2] X[4] X[5] X[6] + X[3] X[4] X[5] X[6] +  
 X[1] X[3] X[4] X[7] + X[2] X[3] X[4] X[7] + X[1] X[3] X[5] X[7] + X[2] X[3] X[5] X[7] +  
 X[1] X[4] X[5] X[7] + X[2] X[4] X[5] X[7] + X[3] X[4] X[5] X[7] + X[1] X[3] X[6] X[7] +  
 X[2] X[3] X[6] X[7] + X[1] X[4] X[6] X[7] + X[2] X[4] X[6] X[7] + X[3] X[4] X[6] X[7]

In[20]:=