Quiz2 - Math 313 **- Fall** 2014

1. (a) Let

$$(d_1, d_2, d_3, \cdots)$$

be an infinite sequence of digits d_i, where a number d_i is said to be a *digit* if it is a natural number between 0 and 9. Given such a sequence consider the infinite series

$$\sum_{k=1}^{\infty} d_k/10^k.$$

We abbreviate this series by

$$.d_1d_2d_3\cdots = \sum_{k=1}^{\infty} d_k/10^k.$$

Note that the partial sums for this series are

$$S_n = \sum_{k=1}^{n} d_k/10^k = .d_1d_2d_3\cdots d_n.$$

Prove that for any infinite sequence of digits, the associated series described above converges to a real number. (Hint: Show that the partial sums form a monotone, bounded sequence. Explain how the Completeness Axiom for the Real Numbers implies that such a sequence must have a limit.)

(b) Give the definition of limit for a sequence of real numbers. Prove that $1 = .9999\cdots$ by using the definition of limit.