HOTEL INFINITY

There was once a hotel in the mountains. So many people liked to go there that the manager decided to extend the hotel.
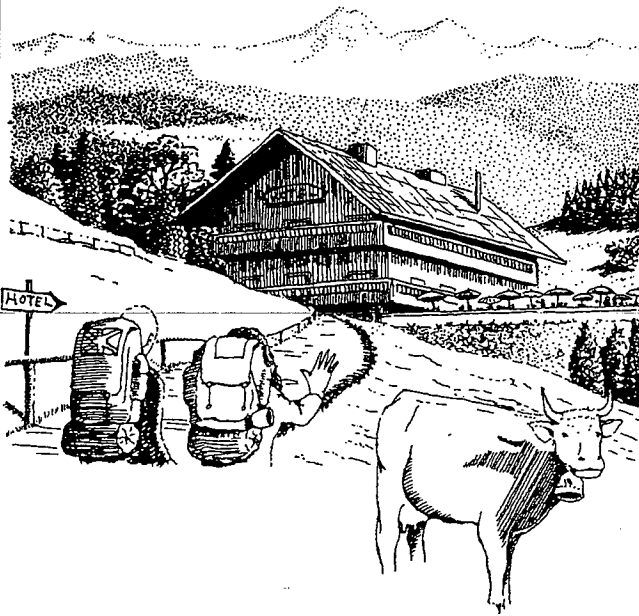
But still it was always full so he continued to add until eventually... it became infinitely large. The manager was very pleased and he especially liked his slogan...
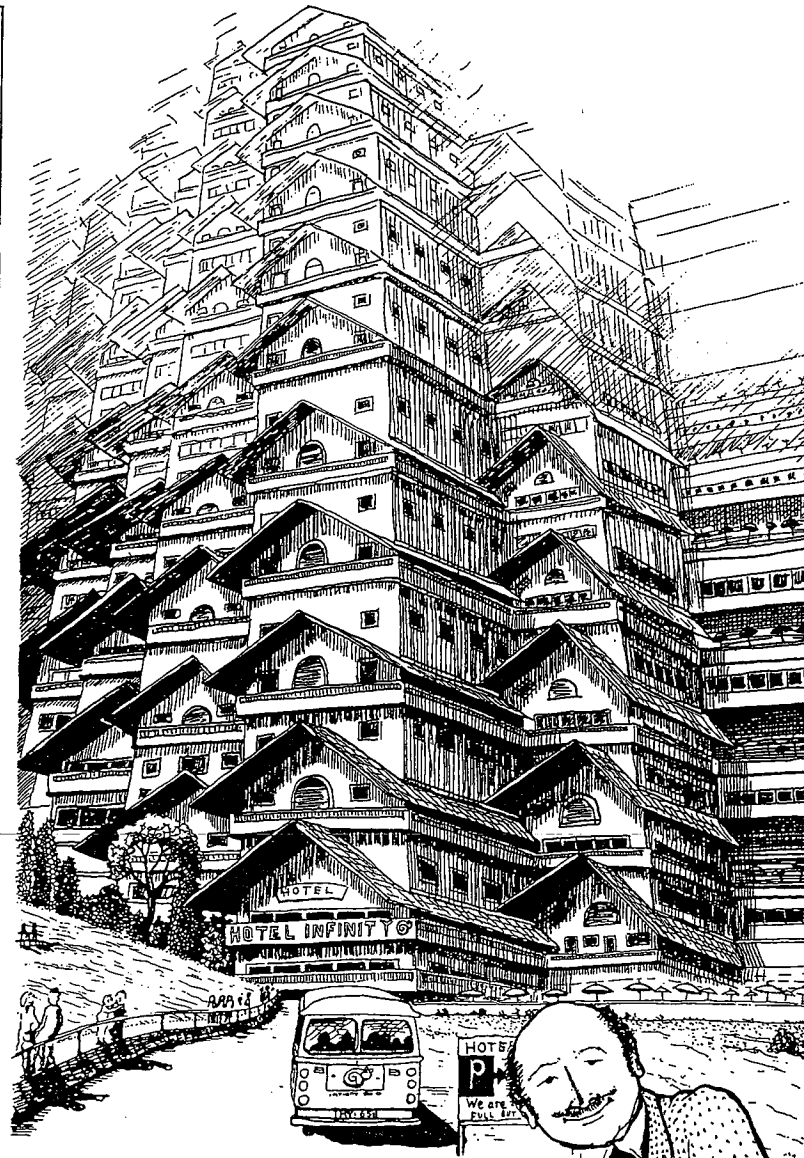
'We are always full – but we always have room for you!'

One day a gentleman came to the hotel asking for a room. The receptionist looked through her books but could not find an empty room in the hotel. But the manager knew what to do. He gave orders for all the guests to be moved into different rooms.
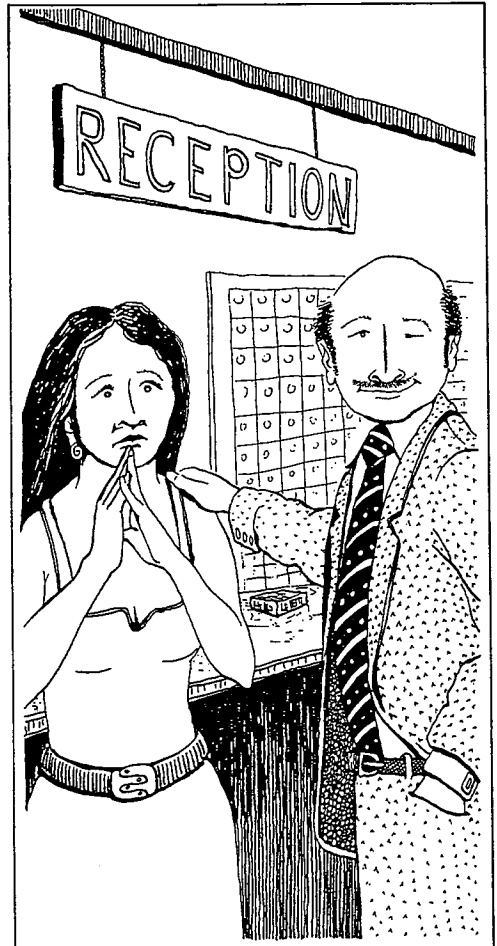
The guest in room 1 was moved to room 2. The guest in room 2 was moved to room 3. The guest in room 3 was moved to room 4 and so on.

Everyone had a room to move to – because although there was an infinite number of guests, there was also an infinite number of rooms. And of course room 1 was then empty and that was given to the new arrival.

The other guests were not very pleased at having to change rooms. And anyway they didn't much like the manager, who seemed a bit of a know-all. So they were very excited the next day when one of the buses from the 'Infinite Coach Hire Company' unloaded its passengers at the hotel.

'We always have room for you' one of them mimicked the manager, and the others laughed.

Seeing this infinite crowd of new guests, Miss Keeper at once called the manager. He thought for a moment and then he told her, 'Move the guest in room 1 into room 2 and the guest in room 2 into room 4 and the guest in room 3 into room 6 and so on. This will leave vacant rooms 1, 3, 5, 7, etc. Put the new arrivals into those rooms.'
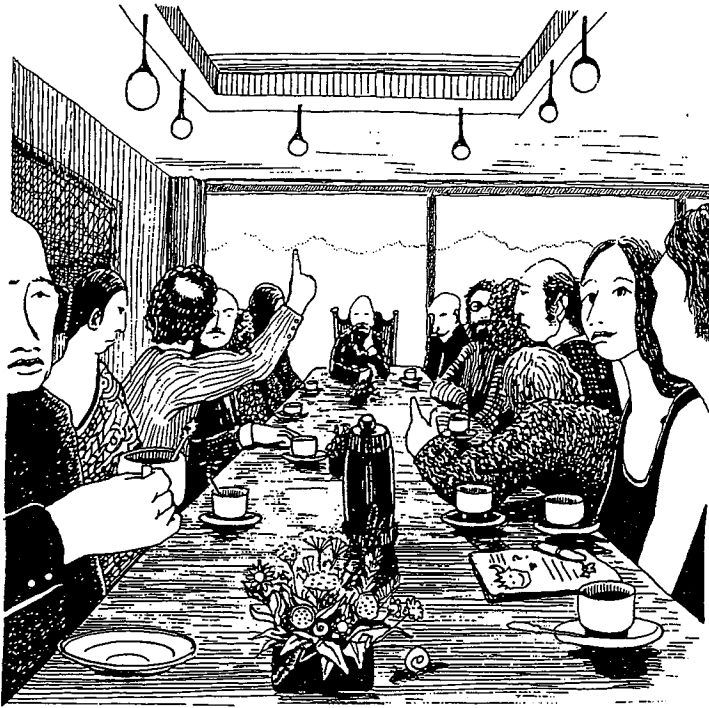
The guests were very upset at having to move rooms again, but they were even more upset that the manager had again found a way of doing the impossible. So they called a meeting to think up an idea that would really beat him. No one could come up with anything that was foolproof until at last the professor had an idea which he thought would work:

The professor's plan

1. During the next few weeks there will be a series of meetings.
2. First there will be an "empty meeting" with no one present.
3. Next, every guest will go to a meeting at which only he, or she, is present.
4. Then the guests will meet in pairs: these meetings must cover <u>all possible</u> pairs of guests.
5. Then there will be meetings of <u>all possible</u> groups of three guests.
6. Then meetings of four guests, then of fives, then of sixes, and so on, until—
7. There have been meetings of <u>all possible groups of all sizes.</u>
8. This series of meetings must be completed by 1st. december.
9. The purpose of each meeting is to invite a guest (who may, or may not be a present guest) to stay at this hotel next Christmas.
10. When a meeting has agreed on its guest, the manager must be asked to reserve a room for that person.
11. No two meetings may invite the same person (a list will be kept to avoid this happening)
12. The professor will decide whom the "empty meeting" (rule 2) will invite, and he will ask the manager to book a room for that guest.

Although the other guests didn't really understand it, they thought it must be a very good plan. The rules were sent to all the guests.

The meetings quickly got under way. Some of the guests wanted to stay on for Christmas and they were 'invited' by one of the meetings. But when he booked the rooms, the manager did not always give these guests the rooms they already occupied. And then one day a member of staff brought him a copy of the professor's plan which she had found whilst cleaning one of the rooms. The plan worried him. That night he couldn't sleep. He went to his office and began to write.

'I'll label each guest with the number of the room they now occupy. So Mr. Zawirski – he's in room 26 – he'll be just '26'. The person in room N will be called 'N'... now these meetings – I'll label them according to the number of the room I give to its guest. The meeting whose guest has Mr. Zawirski's room will be M_{26}'.

'The meeting whose guest will occupy room N will be called M_N. Then he thought about the time when he would have made all the room allocations.

'Some of the people will have been to the meeting whose guest I am putting into the room they now occupy. But I had to give a room (it was 126) to the guest of the empty meeting and person 126 was not at that meeting – no one was.

So there's at least one person who does not go to the meeting whose guest gets that person's room.'

There may be more than one like that. And they will have a meeting (rule 7) – and I shall give that meeting's guest a room – say 314. So it's meeting M_{314}. Now what about person 314?

Was she (or he) at meeting M_{314}? No – because M_{314}'s guest doesn't get the room of anyone at M_{314}.

But M_{314} is a meeting of all the people who did not go to the meeting whose guest is given their own room. That means 314 *would* have gone to M_{314}.

'But that's not possible' the manager cried out. It is saying, 314 is *not* at the meeting means that 314 *is* at the meeting. He (or she) can't be both *at* the meeting and *not at* the meeting.

So I can't give this meeting's guest room 314 or any other room!'

The manager thought all night about this – but he could find no way out. So in the morning he wrote a letter of resignation and left the hotel for good.

The guests watched him go, and then had a little celebration at their victory.

But after a few weeks they found the service at the hotel began to get worse and worse. They had to wait ages for their meals, and sometimes the potatoes ran out altogether.

And there were awful muddles about the bookings for rooms. So they began to feel very sorry that they had been jealous just because the manager was so clever.