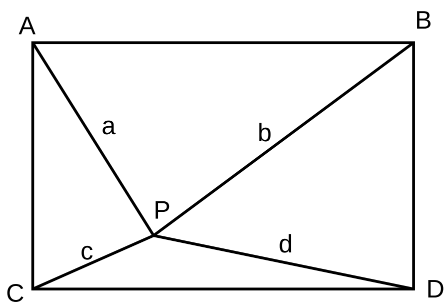
Three Extra Credit Problems For Math 215

1. Consider the rectangle diagram shown in the figure below. The corners of the rectangle are A, B, C, D and there is a point P in the rectangle whose distance from A is a, from B is b, from C is c and from D is d. Prove that

$$a^2 + d^2 = b^2 + c^2.$$



2. Prove that

$$2 = (2 + \frac{10}{\sqrt{27}})^{1/3} + (2 - \frac{10}{\sqrt{27}})^{1/3}.$$

In order to do this, note that if $x = a + b$, then

$$x^3 = a^3 + b^3 + 3ab(a + b).$$

Hence

$$x^3 = a^3 + b^3 + 3abx.$$

Apply this result to the problem.

3. Let S be a subset of $\{1, 2, 3, \cdots, 2n\}$. Suppose that $|S| = n + 1$. Prove that S contains two numbers such that one number divides the other number. (Hint: Any natural number m can be written in the form $m = (2k - 1)2^j$. That is, it can be written in the form of an odd number times a power of two. For such a number m, define $f(m) = k$. Show that this gives a map $f : S \longrightarrow \{1, 2, \cdots, n\}$, and make use of this map.)