A Game Based on Knot Theory

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1. Background — Region Crossing Change

A puzzle game — *Region Select*[a] — was produced by Akio Kawauchi, Kengo Kishimoto and the author at Osaka City University Advanced Mathematical Institute (OCAMI). In December 2011, an Android application "Region Select" was released, and now the number of installations is more than 5,000. (We have a plan for an iPhone version, too.) You can also play Region Select at OCAMI's website [5]. Young and old alike play Region Select all over the world. This game is based on knot theory.

In this report, I will explain how Region Select was created, and what Region Select might do in the future.

In autumn of 2010, Kengo Kishimoto, who was a researcher at OCAMI (and now a lecturer at Osaka Institute of Technology), asked the following question in a seminar at Osaka City University:

Is a region crossing change on knot diagrams an unknotting operation?

A region crossing change is a local move on knot and link diagrams defined by Kishimoto. We explain Kishimoto's question in detail. A knot is an embedding of a circle in the 3-sphere. A knot diagram is a projection on the 2-sphere S^2 of a knot with over/under information such as D and D' in Fig. 1. Note that a knot diagram with n crossings divides S^2 into $n + 2$ parts. We call the parts *regions*. A *region crossing change* on a region R of a knot diagram D is defined to be a crossing change at all the crossings on the boundary of R. Kishimoto's question is whether we can obtain a diagram for the trivial knot from any knot diagram by a finite sequence of region crossing changes. To answer Kishimoto's question, the author, who was a graduate student at Osaka City University, proved the following theorem ([4]):

[a]Region Select is patent pending for the game mechanics and program by Osaka City University.

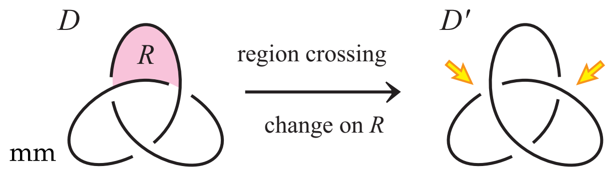

Fig. 1. Region crossing change

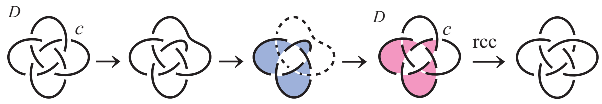

Fig. 2.

Theorem. *We can change any crossing of any knot diagram by a finite number of region crossing changes.*

(See Fig. 2.) It is well-known that we can obtain a diagram of the trivial knot from any knot diagram by crossing changes. Therefore, the answer to Kishimoto's question is "Yes". Recently, Chen and Gao generalised this result to links ([3, 2]).

2. Creation of the Game

From Kishimoto's question, we produced the game as follows: First, we create a *knot projection with lamps*, namely, a knot projection with lamps on the crossings. The lamps can be turned on or off. A region crossing change on a region of a knot projection with lamps will turn on/off of the lamps on the boundary of the region. From the above theorem, we have the following:

We can turn on/off any lamp of any knot projection by a finite number of region crossing changes.

Then, the goal of the game Region Select is to light up all the lamps of a given knot projection with lamps by a sequence of region crossing changes chosen by clicking on regions on a display (see Fig. 3).

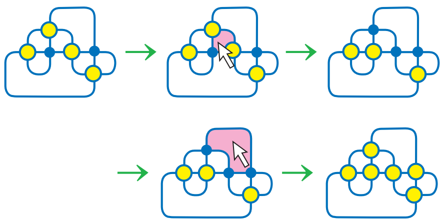

Fig. 3. Region Select

Fig. 4. Region Select for children

Later, Kawauchi, Kishimoto and the author created the dual version of Region Select, and Ahara and Suzuki showed the Region Select with n-colour lamps instead of on/off lamps is also well-defined ([1]). We hope more related games will appear all over the world.

3. Future Development

Since Region Select does not need words or numerical equations, we can imagine many kinds of applications of Region Select. For example, we expect Region Select will be used in primary education. We hope children enjoy this game and appreciate its graphics. Their ability to think ahead will be straightened by playing the game. Now we need to design the game for children as shown in Fig. 4 or [6]. We also expect to use Region Select for training cognitive functions to recognise shape during rehabilitation. Since this game is based on pure mathematics, we expect and believe that Region Select has limitless possibilities.

References

[1] K. Ahara and M. Suzuki, An integral region choice problem on knot projection, arXiv: 1201.4539.

[2] Z. Cheng, When is region crossing change an unknotting operation? arXiv: 1201.1735.

[3] Z. Cheng and H. Gao, On region crossing change and incidence matrix, to appear in *Science China Mathematics*.

[4] A. Shimizu, Region crossing change is an unknotting operation, arXiv:1011.6304.

[5] Region Select: http://www.sci.osaka-cu.ac.jp/math/OCAMI/news/gamehp/etop/gametop.html

[6] Region Select for children: http://www.sci.osaka-cu.ac.jp/math/OCAMI/news/gamehp/c3game/game3/top.html

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