Algebraic Topology – HW#2

I. Exact Couples

Let

$$\begin{tikzcd} D \arrow[rr, "i"] & & D \arrow[dl, "j"] \\ & E \arrow[ul, "k"] & \end{tikzcd}$$

be an exact triangle of modules (so that $\operatorname{Ker} i = \operatorname{Im} k$, $\operatorname{Ker} j = \operatorname{Im} i$, $\operatorname{Ker} k = \operatorname{Im} j$).

Define $d : E \to E$ by $d = jk$.

Thus $d^2 = jkjk = 0$ since $kj = 0$.

Let $H(E) = \operatorname{Ker}(d)/\operatorname{Im}(d)$ be the homology of E with respect to d.

Let $D' = i(D)$ and define a new triangle via

$$\begin{tikzcd} D' \arrow[rr, "i'"] & & D' \arrow[dl, "j'"] \\ & E' \arrow[ul, "k'"] & \end{tikzcd} \qquad \begin{aligned} i' &= i\big|_{i(D)} \\ j'(ix) &= [jx] \\ k'[z] &= kz \end{aligned}$$

where $[\ ]$ denotes homology class.

a) Show that the maps in this <u>derived triangle</u> are well-defined.

b) Show that the derived triangle is exact.

c) Taking the recursive nature of this construction as a subject to investigate, ask questions of your own and explore it in some direction. Report on what you find.

II. Hatcher page 131

1., 2., 4., 8., 11., 12., 15., 17., 20.,
21., 23.

III. Hatcher page 155

2., 3., 28.

IV. (a) Write up our calculation of the homology of real projective spaces, as we did in class (via a cell structure for S^n with two cells in each dimension).

(b) Let P^n denote the n-dimensional real projective space.
Calculate the cohomology groups $H^*(P^n)$, with
(i) integer coeffs.
(ii) $\mathbb{Z}/2\mathbb{Z}$ coeffs.