Algebraic Topology - Spring 2013 - Assignment Number 1.

1. Compute the homology groups of a torus by using a simplicial triangulation of the torus. Specify the triangulation, set up the chain complex, do the calculations. (Hint: Every simplex is a homology between the sum of two of its sides and the remaining side. You can use this to eliminate almost all 1-simplices from consideration.)

2. Recall that the homology groups for a simplicial complex are defined, given an ordering of the 0-simplices of the complex. Prove that the homology groups of the complex are independent of this choice of ordering.

3. A graph is said to be *simple* if it is a one dimensional simplical complex. Thus of any two nodes one can say that there either is one edge joining them, or they are not joined. A spanning tree in a graph G is a connected tree (graph with no cycles) that contains every node of G.

Prove that the first homology group of a simple graph G has rank

$$r(G) = e(G) - t(G)$$

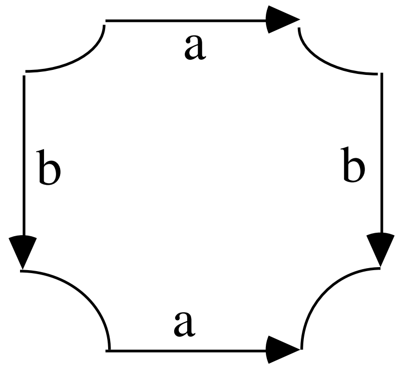
where e(G) is the number of edges of the graph G and t(G) is the number of edges in a spanning tree of G.

Show that for a planar simple graph,

$$r(G) = f(G) - 1$$

where f(G) is the number of faces of the graph G = number of regions in the plane created by the embedding of G in the plane.

4. Let Punc(T) denote the CW complex for the punctured torus, given by the identification space below. Find compatible orientations for all cells in the complex so that the iterated boundary mapping in the chain complex is zero. Compute the homology groups of Punc(T).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